Mathematical Statistics
Test 1
Spring 2006
Name:…….

16+(4+4+4+4+2)+20+24+(4+4+4+2)+4+4=100

1 Let the joint p.d.f. of X and Y be defined by $f(x,y)=c,\ x^2 \le y \le 1,\ 0 \le x \le 1$. Find

(a) the value of c.

(b) $f_1(x)$, the marginal p.d.f. of X.

(c) $f_2(x)$, the marginal p.d.f. of Y.

(d) $P(X \ge 0.5, Y \ge 0.5)$.

2 Assume that X and Y have a bivariate normal distribution with $\mu_X = 22.7$, $\sigma_X^2 = 17.64$, $\mu_Y = 22.7$, $\sigma_Y^2 = 12.25$ and $\rho = 0.78$. Find

(a) $P(18.5 < Y < 25.5)$.

(b) $E(Y \mid X)$.

(c) $Var(Y \mid X)$.

(d) $P(18.5 < Y < 25.5 \mid X = 23)$.

(e) Are X and Y independent? Explain.

3 Suppose X and Y are continuous random variables with joint p.d.f. $f(x,y) = 60x^2 y$ for $x > 0$, $y > 0$, $x + y < 1$, and zero otherwise. Find the following:

(a) Marginal distribution of X.
(b) Conditional p.d.f. of Y given X.
(c) $P(Y > 0.1 | X = 0.5)$.
(d) $E(Y | X = x)$.
(e) $Var(Y | X = x)$.

4 Suppose X and Y are continuous random variables with joint p.d.f. $f(x,y)=(x+y),\ 0<x<1,\ 0<y<1$, and zero otherwise. Find each of the following: **(Use symmetry to save time)**

(a) $f_1(x)$ and $f_2(y)$.

(b) μ_x and μ_y.

(c) σ_x^2 and σ_y^2.

(d) $E(XY)$.

(e) $Cov(X,Y)$.

(f) ρ.

5 Suppose that the random variables X and Y have the following joint p.d.f.:

$$f(x, y) = 4xy, \text{ for } 0 \le x \le 1,\ 0 \le y \le 1.$$

Also let $U = X$ and $V = XY$.

(a) Draw the support of X and Y, and that of U and V.
(b) Determine the joint p.d.f of U and V.
(c) Find the marginal distributions of U and V. (Marginal distribution of V may look a little strange.)
(d) Are U and V independent?

You may give up points and ask me to show you how to draw the support of U and V.

6 X and Y are independent random variables with common m.g.f.. function $M(t) = \exp\left(\frac{t^2}{2}\right)$. Let $W = X + Y$ and $Z = Y - X$. Determine the joint m.g.f $M_{W,Z}(t_1, t_2)$ of W and Z.

7 Let $U_1 \sim \chi^2_{(5)}$ and $U_2 \sim \chi^2_{(3)}$ are two independent χ^2 random variables with respective degrees of freedom 5 and 3. Define a random variable which has a F distribution. What are the numerator and denominator degrees freedom?

RV:

Numerator d.f.

Denominator d.f.