Mathematical statistics
Spring 2006
Test 2
Closed book 50%
Name:……..

1 Let $f(x)=\frac{1}{\theta}e^{-\frac{x}{\theta}}$, $x>0$, $\theta>0$. Let X_1, X_2,…, X_n denote a random sample of size n from this distribution.

(a) What is the parameter space Ω of this distribution?
(b) Find the maximum likelihood estimator (MLE) of θ.
(c) Find the method of moments (MOM) estimator of θ. {Note: $E(X)=\theta$.}
(d) Is the MLE of θ unbiased? (Show your work.)
(e) Find $P(X>1)$ and the MLE of $P(X>1)$.

2 Consider a random sample $\left(X_1, X_2, ..., X_n\right)$ from an exponential distribution with mean θ.

(a) Show that $S = \sum_{i=1}^{n} X_i$ is a sufficient statistics for θ.

(b) Show that the distribution of $S = \sum_{i=1}^{n} X_i$ is a Gamma$\left(n,\ \theta\right)$ distribution.

(Hint: Use m.g.f.)

(c) Find the conditional distribution of $\left(X_1, X_2, ..., X_n\right)$ given S.

(You can do part c even if you can not do part b.)

3 Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from $N(\mu, \sigma^2)$ where σ is unknown. Prove that $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{(n-1)}$. Derive a 100(1-$\alpha$)% symmetric confidence interval for the population mean μ.

Hint: $\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \dfrac{S}{\sqrt{n}}$

4 Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from $Uni(\theta, 2)$.

(a) Find the MLE of θ.

(b) Find the distribution of $y_1 = \min(X_1, X_2, ..., X_n)$.

(c) Is the MLE of θ unbiased? Show your work.

Hint: $x(2-x)^{n-1} = -(2-x-2)(2-x)^{n-1} = 2(2-x)^{n-1} - (2-x)^n$

5 Consider a random sample $\left(X_1, X_2, ..., X_n\right)$ from a $Geo(\theta)$.

(a) If $E(X_i) = \frac{1}{\theta}$ for $i = 1, 2, \ldots, n$, than show that $\bar{X}$ is an unbiased estimator for $\tau(\theta) = \frac{1}{\theta}$.

(b) If $Var(X_i) = \frac{1-\theta}{\theta^2}$ for $i = 1, 2, \ldots, n$, then show that $Var(\bar{X}) = \frac{1-\theta}{n\theta^2}$.

(c) Find the Cremer-Rao lower bound for the variance of any unbiased estimator of $\tau(\theta)$.

(d) Is $\bar{X}$ UMVUE of $\tau(\theta)$? Explain.

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6	6.4.6
7	6.4.8
8	6.5.2
9	6.5.12
10	6.6.2
11	6.6.8
12	6.7.4
13	6.7.14
14	6.8.4
15	6.8.8