Mathematical Statistics
Spring 2003
Test1

Name:……………………………..

1 Let the joint p.m.f of X and Y be given by $f(x, y) = \frac{x y^2}{30}$, for x = 1, 2, 3 and y = 1, 2 and zero otherwise. **(This is a discrete distribution)**

(a) Construct a probability table.
(b) Find the marginal distribution of X.
(c) Find the marginal distribution of Y.
(d) Are X and Y independent? Explain.
(e) Find $P(X+Y>3)$.

2 Suppose X and Y are continuous random variables with joint p.d.f. $f(x,y) = 60x^2y$ for $x > 0$, $y > 0$, $x + y < 1$, and zero otherwise. Find the following:

(a) Marginal distribution of X.

(b) Conditional p.d.f. of Y given X.

(c) $P(Y > 0.1 | X = 0.5)$.

(d) $E(Y | X = x)$.

(e) $Var(Y | X = x)$.

3 Suppose X and Y are continuous random variables with joint p.d.f. $f(x,y) = c(x+y)$, $0 < x < 1$, $0 < y < 1$, and zero otherwise. Find each of the following: **(Use symmetry to save time)**

(a) Evaluate the constant c.

(b) $f_1(x)$ and $f_2(y)$.

(c) m_x and m_y.

(d) s_x^2 and s_y^2.

(e) $E(XY)$.

(f) $Cov(X,Y)$.

(g) r.

4 Assume that X and Y have a bivariate normal distribution with, $m_X = 70$, $s_X^2 = 100$, $m_Y = 80$, $s_Y^2 = 169$ and $r = 0.4$. Find the following:

(a) $P(Y < 100)$.

(b) $E(Y | X = x)$.

(c) $Var(Y | X = x)$.

(d) $P(Y < 100 | X = 72)$.

(e) Are X and Y independent?

5 Suppose that the random variables X and Y have the following joint p.d.f.:

$$f(x, y) = 4xy, \text{ for } 0 \le x \le 1,\ 0 \le y \le 1.$$

Also let $U = X$ and $V = XY$.

(a) Draw the support of X and Y, and that of U and V.
(b) Determine the joint p.d.f of U and V.
(c) Find the marginal distributions of U and V. (Marginal distribution of V may look a little strange.)
(d) Are U and V independent?

Take home problem (10 points)

6 Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistic of n independent observations from $U(0, a)$ distribution. Find the following:

(a) The p.d.f. of Y_1.

(b) The p.d.f. of Y_n.

(c) $E(Y_1)$.

(d) $E(Y_n)$.

(e) Is $E(Y_n) = a$? (If $E(Y_n) = a$, Y_n is called an unbiased estimator for a.)

(f) If $E(Y_n) \neq a$, find $b(Y_n) = E(Y_n) - a$. ($b(Y_n)$ is called the bias of the estimator)

(g) Find $\lim_{n\to\infty} E(Y_n)$ and $\lim_{n\to\infty} b(Y_n)$. (If $\lim_{n\to\infty} E(Y_n) = a$, or equivalently $\lim_{n\to\infty} b(Y_n) = 0$, Y_n is called an asymptotically unbiased estimator for a.)