Mathematical Statistics
Test 1
Spring 2005
Name:…….
15+16+10+20+20+5+5+5+5

1 Let the joint p.d.f. of X and Y be defined by $f(x,y)=\frac{x+y}{12}$, x =1, 2, y = 1, 2.

Find

(a) $f_1(x)$ and $f_2(x)$, the marginal p.d.f.'s of X and Y .

(b) μ_X and μ_Y.

(c) σ_X^2 and σ_Y^2.

(d) $Cov(X,Y)$.

(e) ρ.

(Use symmetry)

2 Suppose that the random variables X and Y have the following joint p.d.f.:
$f(x, y) = 8xy$ for $0 \le x \le y \le 1$.

Also let $U = \frac{X}{Y}$ and $V = Y$.

(a) Draw the support of X and Y, and that of U and V.
(b) Determine the joint p.d.f of U and V.
(c) Find the marginal distributions of U and V.
(d) Are U and V independent?

3 Suppose that the joint p.d.f. of two random variables X and Y is as follows:

$$f(x,y)=\begin{cases} c\left(x+y^2\right) & \textit{for } 0\le x\le 1 \textit{ and } 0\le y\le 1, \\ 0 & \textit{otherwise.} \end{cases}$$

Find

(a) the conditional p.d.f. of X for any given value of Y.

(b) $P\left(X<0.5\,|\,Y=0.5\right)$.

4 Assume that X and Y have a bivariate normal distribution with, $\mu_X = 10$, $\sigma_X^2 = 16$, $\mu_Y = 12$, $\sigma_Y^2 = 9$ and $\rho = 0.8$. Find

(a) $P(9 < Y < 17.5)$.

(b) $E(Y \mid X)$.

(c) $Var(Y \mid X)$.

(d) $P(9 < Y < 17.5 \mid X = 11)$.

(e) $f_X(x)$.

5 Let the joint p.d.f. of X and Y be defined by $f(x, y) = 2$, $0 \le x \le y \le 1$. Find

(a) $f_1(x)$, the marginal p.d.f. of X.

(b) $h(y \mid x)$, the conditional distribution of Y given $X = x$.

(c) $E(Y \mid X = x)$, the conditional mean of Y given $X = x$.

(d) If it is given that $\sigma_X^2 = \sigma_Y^2$, find the value of ρ. (Hint: If $E(Y \mid X = x)$ is linear, it is the least squares regression line.)

6 P rove that $Var(Y) = E\left[Var(Y \mid X)\right] + Var\left[E(Y \mid X)\right]$.

7 Let X and Y be random variables with joint moment generating function $M_{X,Y}(t_1, t_2) = 0.3 + 0.1e^{t_1} + 0.2e^{t_2} + 0.4e^{t_1+t_2}$. What is $E(2X - Y)$?

8 Let $U_1 \sim \chi^2_{(5)}$ and $U_2 \sim \chi^2_{(3)}$ are two independent χ^2 random variables with respective degrees of freedom 5 and 3. Let $V = \frac{U_1}{U_2}$. Find c such that $P(V \geq c) = 0.05$.

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9 Let X_1, X_2, and X_3 be a random sample from $Exp(1)$. Find the mean of the smallest order statistic.