Mathematical Statistics
Test 2
Spring 2003
Name:………………………………………

Closed book part
40+10 points

1   Consider the probability density function $f(x;q) = \dfrac{1}{q^2} x e^{-x/q}, x \geq 0, q > 0$. Let

$X_1, X_2, \ldots, X_n$ be a random sample from $f(x;q)$.

    (a)   Show that $E(X) = 2q$.

    (b)   Find the method of moment (M.O.M.) estimator of q.

    (c)   Find the maximum likelihood estimator (M.L.E.) of q.

    (d)   Is the M.L.E. of q unbiased? Show your work.

    (e)   Find the M.L.E. of $t(q) = \int_0^\infty x^2 f(x)\,dx$. First evaluate $t(q)$.

2   Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from $N(m, s^2)$ where s

is unknown. Using the fact that $\dfrac{(n-1)S^2}{s^2} \sim c^2_{(n-1)}$, derive a symmetric two-

sided confidence interval for s^2.

Open textbook and class notes part
48 points

1   7.2.12
2   7.3.2
3   7.4.6 (b) and (c)   (Use the result of part (a).)
4   7.4.8
5   7.5.4
6   7.5.14
7   7.6.2
8   7.6.10

Take home
4 points
1   7.4.6 (a)