Mathematical Statistics
Spring 2005
Test 2
04/07/05
Closed book part
Name:…….
10+15+10+12+3 = 50

1 Let $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from this distribution.

(a) Find the maximum likelihood estimator (MLE) of θ.
(b) Find the method of moments (MOM) estimator of θ.

2 Let $f(x) = \frac{1}{\theta^2} x e^{-x/\theta}$, $x > 0$, $\theta > 0$. Let $X_1, X_2, \ldots, X_n$ be a random sample from this distribution. Notice that $X \sim Gamma(2, \theta)$.

(a) Find the maximum likelihood estimator (MLE) of θ.
(b) Find the method of moments estimator of θ.
(c) Is the MLE of θ unbiased? Show your work.

3 Let $f(x) = \frac{1}{\theta} e^{-\frac{(x-\eta)}{\theta}}$ for $x > \eta$.

(a) Find the MLE of θ and η.
(b) Is the MLE of η unbiased? Show your work.

4 Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from $N(\mu, \sigma^2)$. Derive a 100(1-α)% symmetric confidence interval (C.I.) for the population mean μ assuming σ is unknown.

5 Explain the invariance property.

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Test 2 (Take home)
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50 points
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1 7.2.12
2 7.3.6 (a)
3 7.3.12 (a) & (b)
4 7.4.9
5 7.5.14
6 7.8.2 (a), find s^2, and then a 90% confidence interval for β. You may use EXCEL.