Mathematical Statistics
Spring 2003
Test 4 (Closed Book)
Name:……

1 Consider a random sample $(X_1, X_2, ..., X_n)$ from a Poisson distribution with mean l.

(a) Show that $S = \sum_{i=1}^{n} X_i$ is a sufficient statistics for l.

(b) Show that the distribution of $S = \sum_{i=1}^{n} X_i$ is also a Poisson distribution.

(Hint: Use m.g.f.)

(c) Show that the conditional distribution of $(X_1, X_2, ..., X_n)$ given S is independent of l.

(You can do part c even if you can not do part b.)

2 Consider a random sample $\left(X_1, X_2, ..., X_n\right)$ from a $N(m, 81)$.

(a) Show that $C = \{(x_1, x_2, ..., x_n) : \bar{x} \geq c\}$ is a best critical region for testing $H_o : m = 40$ against $H_a : m = 44$.

(b) Find n and c such that $a = 0.05$ and $b = 0.10$.

3 Consider a random sample $\left(X_1, X_2, ..., X_n\right)$ from a $N\left(m, 100\right)$.

(a) To test $H_o : m = 75$ against $H_a : m > 75$, what is the critical region specified by the likelihood ratio test criterion?

(b) Is this test uniformly most powerful? Explain.

(c) Can H_o be rejected at 0.05 level of significance if a random sample of size 25 yielded $\overline{X} = 78.6$?

(d) What is the p-value of this test?

4 Consider a random sample $(X_1, X_2, ..., X_n)$ from a $b(1, p)$ and let $t(p) = p$.

(a) If $E(X_i) = p$ for $i = 1, 2, \ldots, n$, than show that $\bar{X}$ is an unbiased estimator for $t(p)$.

(b) If $Var(X_i) = p(1-p)$ for $i = 1, 2, \ldots, n$, then show that $Var(\bar{X}) = \frac{p(1-p)}{n}$.

(c) Find the Cremer-Rao lower bound for the variance of any unbiased estimator of $t(p)$.

(d) Is $\bar{X}$ UMVUE of $t(p)$? Explain.

5 Let $X \sim N(m,100)$. To test $H_o : m = 80$ against $H_a : m > 80$, let the critical region be defined by $C = \{(x_1, x_2, ..., x_{25}) : \bar{x} \geq 83\}$, where $\bar{x}$ is the sample mean of a random sample of size 25 from this distribution.

(a) What is the power function $K(m)$ of this test?

(b) What is the significant level of this test?

(c) What are the values of $K(80)$, $K(83)$, and $K(86)$?

(d) What is the p-value corresponding to $\bar{x}$ =83.41?