Probability and Statistics
Spring 2008
Name: $\qquad$
$6+12+6+12+12+10+8+12+6+8+8=100$

1. In how many different ways can seven businessmen attending a convention be assigned to one triple and two double hotel rooms?
2. Let $P(A)=0.5, P(B)=0.7$, and $P\left(A^{\prime} \cap B^{\prime}\right)=0.1$. Find the following:
a. $\quad P(A \cup B)$
b. $\quad P(A \cap B)$
c. $P\left(A \cup B^{\prime}\right)$
3. Prove that $P(A \cap B \cap C)=P(A) P(B \mid A) P(C \mid A \cap B)$. Start from the right hand side and use the definition of conditional probability.
4. If $A$ and $B$ are independent. Then prove that $A$ and $B^{\prime}$ are also independent.
5. Suppose a company rents cars from three rental agencies: $50 \%$ from agency $1,30 \%$ from agency 2 , and $20 \%$ form agency 3 . Assume $5 \%$ of the cars from agency $1,8 \%$ of cars from agency 2 , and $10 \%$ of the cars from agency 3 smells inside.
a. What is the probability a rental car deliver will smell?
b. If a given car smells inside, what is the probability that the car came from agency1?
6. Suppose that the probability that any particle emitted by a radioactive material will penetrate a certain shield is 0.001 . If ten particles are emitted, what is the probability that exactly one of the particles will penetrate the shield?
7. A hand of 13 cards is to be dealt at random and without replacement from an ordinary deck of playing cards. Find the conditional probability that there are exactly four kings in the hand given that the hand contains at least three kings.
8. Let $P(A)=0.6, P(B)=0.7$, and $A$ and $B$ are independent. Find the following:
a. $P(A \cap B)$
b. $P(A \cup B)$
c. $P\left(A \cup B^{\prime}\right)$
d. $P(A \mid B)$.
9. A travel brochure lists 10 museums in the city of London. In how many ways can a tourist visit four museums?
(a) if the order in which the museums are visited does not mater;
(b) if the order in which the museums are visited does matter?
10. If $P(A)=0.4, P(B)=0.3$ and $A$ and $B$ are mutually exclusive, find $P\left(A \cap B^{\prime}\right)$ and $P\left(A^{\prime} \cup B^{\prime}\right)$.
11. If $P(A)=0.6, P(B)=0.8$ and $A \subset B$, find $P\left(A \cap B^{\prime}\right)$ and $P\left(A^{\prime} \cup B^{\prime}\right)$.
