Probability and Statistics
Test 2
Spring 2002

Name:…….
Problems in each page is worth 20 points.

1 Let $f(x) = cx^2$ where x = -5, -3, -1, 1, 3, 5 and $A = \{1,3\}$.

(a) Find the value of c.

(b) Find $P(A)$.

2 In a lot of 50 light bulbs, there are 3 defective bulbs. An inspector inspects 6 bulbs selected randomly. Find the probability of finding at least two defective bulbs. **(Setup do not simplify)**

3 Let $f(x) = \frac{4-x}{6}$ for x = 1, 2, 3. Find the following:

(a) $E(X)$.

(b) $E\left(X^2\right)$.

(c) $E\left(2X^2 + 3X + 4\right)$.

(d) $Var(X)$.

(e) $Var(2X + 3)$.

4 According to Des Moines Register poll, 40% of Iowa farmers support an independent Palestinian state. Let X be the number of Iowa farmers out of a random sample 20 who support an independent Palestinian state.

(a) Find the mean and variance of X.

(b) Find $P(X \geq 7)$.

(c) Find $P(X = 5)$.

5 Derive the moment generating function of **one** of the following distributions.

(a) Binomial. (b) Geometric. (c) Poisson.

6 If X have a Poisson distribution so that $2P(X=2)=2P(X=0)+P(X=1)$, find $P(X=3)$.

7 If the moment generating function of X is $M_X(t) = \frac{1}{4}e^{-t} + \frac{2}{4} + \frac{1}{4}e^{t}$, find the following:

(a) $f(x)$.

(b) $M_X^{(1)}(t)$.

(c) $M_X^{(2)}(t)$.

(d) $M_X^{(3)}(t)$.

(e) $M_X^{(4)}(t)$.

(f) $E\left(X^{2m}\right)$.

(g) $E\left(X^{2m+1}\right)$.

(h) $Var\left(X^{5}\right)$.

8 A certain type of aluminum screen that is two feet wide has on the average one flaw in a 200-foot roll. Find the probability that a 50-foot roll has no flaws.

9 Let X equal the number of rolls of a balanced six-sided die that are required to observe the first six on the top.

(a) Find the p.m.f. of X.
(b) Give the values of the mean, variance, and standard deviation of X.
(c) Find $P(X \leq 2)$ and $P(X > 20)$.

10 Prove one of the following.

(a) $Var(aX + b) = a^2 Var(X)$
(b) If $R(t) = \ln\{M_X(t)\}$, than $R^{(1)}(0) = m$ and $R^{(2)}(0) = s^2$.