

Probability and Statistics
Test 2

Name:.....

$$6+14+10+8+12+6+10+10+12+10+5=103$$

1. Fill in the following blanks:

The probability mass function of a random variable X is a function that satisfies the following properties:

a.

b.

c.

2. Find the value of the constant c .

a. if $f(x) = c\left(\frac{1}{2}\right)^x$, $x = 5, 6, 7, \dots$

b. if $f(x) = c\left(\frac{1}{2}\right)^x$, $x = 5, 6, 7, \dots, 25$

3. Suppose that there are 5 white balls and 3 black balls in a container. Select one randomly and note the color. Without replacing, select another one and note the color. Draw a tree diagram to represent this experiment and give the probabilities of each possibility. What is the probability of getting one white ball and one black ball?

4. Prove that $\text{Var}(aX + b) = a^2\text{Var}(X)$.

5. Let $f(x) = \frac{x}{10}$ for $x=1, 2, 3, 4$. Find the followings:

a. $E(X)$

b. $Var(X)$

c. $E\{[X - E(X)]^2\}$

d. $Var(2X + 1)$

6. Let the random variable X have a Geometric distribution with variance 20. Find $P(X \geq 2)$.

7. Let $f(x) = q^{x-1}p$; $x=1,2,\dots$, where $q=1-p$. Prove that $E(X) = \frac{1}{p}$.
8. Suppose a basketball player can make a free throw 80% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots.
- Find the mean and variance of X .
 - Find $P(X = 15)$.
9. The American Almanac of Jobs and Salaries, reported that 30% of accountants are employed in public accounting. Assume that this percentage applies to a group of 10 college graduates just entering the accounting profession.
- Find the probability that at least 3 graduates will be employed in public accounting.
 - Find the probability that at most 3 graduates will be employed in public accounting.
 - Find the probability that less than 3 graduates will be employed in public accounting.
 - Find the probability that more than 3 graduates will be employed in public accounting.

10. If the moment generating function is $M_X(t) = \exp\{4(e^t - 1)\}$, then find the mean and variance of X .

11. If $M_X(t) = 0.5e^{-t} + 0.5e^t$, then show that $E(X^r) = 0$ when r is odd and $E(X^r) = 1$ when r is even.