Probability and Statistics
Test 3
Spring 2008

Name:…….

1 Let $f(x)=\begin{cases}-cx & \text{for } -2<x<0\\ cx & \text{for } 0\le x<2\end{cases}$. Find the value of $c$.

2 Let $f(x)=cx^3e^{-x^4}$, $0<x<\infty$. Find the value of $c$.

3 Let $F(x)=\begin{cases}\left(1-e^{-x^2}\right) & \text{if } x\ge 0\\ 0 & \text{if } x<0\end{cases}$, Find $f(x)$.

4 Let $f(x)=\frac{50}{x^3}$, $x>5$. Find the median $(\pi_{0.64})$ of the distribution.

5 Let the probability density function of $X$ be $f(x)=\frac{1}{16}x^2e^{-x/2}$, $x\geq 0$. Find the probability density function of $Y=\sqrt{X}$.

6 Let the p.m.f. of $X$ be given by the following table.

| $X$ | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $f(x)$ | 0.25 | 0.25 | 0.25 | 0.25 |

Find the p.d.f of $Y=X^2$.

7 Let $f(x)=\frac{1}{25}e^{-\frac{x}{25}}$, $0<x<\infty$. Let $A=(50,100)$. Find $P(A)$.

8 Let $f(x)=\lambda e^{-\lambda x}$, $0<x<\infty$. Prove that $P(X>i+j \mid X>j)=P(X>i)$.

9 Let $f(x)=\begin{cases} -x, -1<x<0 \\ x,\ 0<x<1 \\ 0,\quad otherwise \end{cases}$. Find the c.d.f. of $X$, $F(x)$.

10 Let $X_1$ and $X_2$ be two independent random variables with probability density functions $f_1(x_1)=0.5,\ 0<x_1<2$ and $f_2(x_2)=2x_2,\ 0<x_2<1$. Find the following:

(a) $P(0<X_1<0.5,\ 0.5<X_2<1)$.

(b) $P(0<X_1<2,\ 0.25<X_2<median)$.

11 Let $X_1$, $X_2$, and $X_3$ be a random sample of size three from $f(x) = 3x^2$ for $0 < x < 1$. Find the mean and variance of $Y = 3X_1 + 2X_2 + X_3$.

12 Let $X_1$, $X_2$, and $X_3$ are i.i.d. $Exp(2)$. Find the distribution of $Y = X_1 + X_2 + X_3$.