

Probability and Statistics  
Test 1  
Spring 2005

Name:.....  
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- 1 (a) Prove the following theorem.  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
Give a set theoretic proof. An argument to justify it will get only 2 points out of 6.
- (b) In a certain town 60% of the population reads the morning news paper, 30% of the people reads the afternoon newspaper, and 18% reads both news papers. What percentage of them reads at least one newspaper?

- 2 If  $P(A) = 0.45$ ,  $P(B) = 0.52$ , and  $P(A \cap B') = 0.28$ , then find
- (a)  $P(A \cap B)$
- (b)  $P(A \cup B)$
- (c)  $P(A' \cup B')$
- (d)  $P(A' \cup B)$

- 3
- (a) A carton of 12 batteries contains 4 that are defective. In how many different ways can one choose 3 of these batteries so that exactly one of the defective batteries is included?
  - (b) There are 20 members in a club. In how many different ways can they choose a president, secretary, and a treasurer?
  - (c) How many different varieties of pizza can be made if you have the following choices: 3 different sizes, 3 different crusts, and 4 toppings from which you can select from 0 to 4 (cheese is automatic)?

4 If  $P(A) = 0.40$ ,  $P(B) = 0.55$ , and  $P(A \cup B) = 0.80$ , then find the following.

- (a)  $P(A|B)$
- (b)  $P(A'|B)$
- (c)  $P(A|B')$
- (d)  $P(A'|B')$ .

5 Let  $P(A) = 0.60$  and  $P(B) = 0.30$ . If  $A$  and  $B$  are independent then find the following.

(a)  $P(A \cup B)$

(b)  $P(A \cap B')$

6 Let  $P(A) = 0.50$  and  $P(B) = 0.30$ . If  $A$  and  $B$  are mutually exclusive then find the following.

(a)  $P(A \cup B)$

(b)  $P(A \cap B')$

7 If  $P(A) = 0.5$ ,  $P(B) = 0.8$  and  $A \subset B$ , find the following.

(a)  $P(A' \cap B)$

(b)  $P(A' \cup B')$ .

- 8 Suppose  $A$ ,  $B$ , and  $C$  are mutually independent events and that  $P(A) = 0.6$ ,  $P(B) = 0.8$ , and  $P(C) = 0.9$ . Find the probability that exactly two of the three events occur.
- 9 Bean seed from supplier A have a 90% germination rate and those from supplier B have an 80% germination rate. A seed packing company purchases 45% of their bean seeds from supplier A and 55% from supplier B and mixes these seeds together.
- (a) Find the probability that a seed selected at random from the mixed seeds will germinate, say  $P(G)$ .
- (b) Given that a seed germinates, find the probability that the seed was purchased from supplier B.
- 10 A box contains three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. One card is selected from the box at random, and the color on one side is observed. If this side is green, what is the probability that the other side of the card is also green?