

Probability and Statistics  
Spring 2006  
Test1

Name:.....

$$24+8+8+8+10+10+4+4+8+10+6 = 100$$

1 Let  $P(A) = 0.45$ ,  $P(B) = 0.65$ , and  $P(A \cap B) = 0.25$ . Find the following:

(a)  $P(A \cup B)$ .

(b)  $P(A' \cup B')$

(c)  $P(A \cap B')$

(d)  $P(A' \cup B)$

(e)  $P(B|A)$ .

(f)  $P(A \cap B | A \cup B)$ . First find the set  $(A \cap B) \cap (A \cup B)$ .

2 Let  $P(B) = 0.50$  and  $P(A \cap B) = 0.20$ . Assuming  $A$  and  $B$  are independent, find  $P(A \cup B)$ .

3 Let  $P(A) = 0.45$ ,  $P(B) = 0.25$ , and  $A$  and  $B$  are mutually exclusive. Find  $P(A' \cap B')$ .

- 4 Give a **set theoretic** proof to **one** of the following theorems.
- (a) If  $A$  and  $B$  are any two events, then  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .
- (b) If  $A$  and  $B$  are independent, then so are  $A$  and  $B'$ .  
(Partial credit will be given to a non set theoretic proof)
- 5 Suppose that 12 six-sided balanced dice are to be rolled. Find the probability that each of the six numbers will appear twice. Simplify.
- 6 Suppose that Pete Sumpras and Andre Agassi are playing a tennis match in which the first player to win three sets wins the match.
- (a) In how many different ways could this match end?
- (b) If the probability that Andre will win any particular set against Pete is 0.48, what is the probability that Andre will win the match?

7 Let  $A, B, \dots, J$  are 10 students in a class. Suppose you want to select 3 students for a committee. Also you want to include  $A$  in this group. In how many different ways can you do this?

8 Let  $A, B,$  and  $C$  are mutually independent events.

(a) Write down the four equations equivalent to the above statement.

9 Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of getting a heart on the first draw and a face card (king, queen, or jack) on the second draw.

10 In a certain city, 30% of the people are Conservatives, 50% are Liberals, and 20% are Independents. Records show that in a particular election, 65% of the Conservatives voted, 82% of the Liberals voted, and 50% of the Independents voted.

- (a) If a person in the city who is eligible to vote is selected at random what is the probability that s/he did not vote?
- (b) If a person in the city who is eligible to vote is selected at random and it is learned that s/he did not vote in the last election, what is the probability that s/he is a Liberal?

11 A drawer contains 4 black socks and an unknown number of white socks. Two socks are selected randomly one at a time. If the probability that two socks are of same color is equal to  $\frac{7}{15}$ , set up an equation to find the number of white socks in the drawer.

Let the number of white socks be  $x$ .