

Probability and Statistics
Spring 2007
Test 1

Name:.....

$$6+8+16+4+10+10+6+10+12+8+10=100$$

- 1 (a) Suppose that someone wants to go by car, by train, or by plane on a week's vacation to one of the five East North Central States. Find the number of different ways in which this can be done.
- (b) How many different ways can the five starting players of a basketball team be introduced to the public?
- (c) Four names are drawn from among the 24 members of a club for the officers of president, vice president, treasurer, and secretary. In how many different ways can this be done?
- 2 A box contains 24 light bulbs of which 4 are defective. One person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs.
- (a) In how many different ways can the first person select all 4 defective light bulbs among his 10?
- (b) In how many different ways can the second person select all 4 defective light bulbs among his 14?
- (c) In how many different ways can the first one have 10 bulbs and the second have 14 bulbs regardless whether the bulbs are defective or not?
- (d) What is the probability that all 4 defective bulbs will be obtained by the same person? (Either the first one or the second one)

3 Consider the experiment of rolling a balanced six-sided die. The sample space can be given as $S = \{1, 2, 3, 4, 5, 6\}$. Let $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$, and $C = \{1\}$. Find the following probabilities.

(a) $P(A \cap B)$

(b) $P(A \cup B)$

(c) $P(A \cup C)$

(d) $P(A' \cap B')$

(e) $P(A' \cap C')$

(f) $P(A' \cap B)$

(g) $P(A' \cup B)$

(i) $P(A \cup B \cup C)$

4 The probability of surviving a certain transplant operation is 0.55. If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.20. What is the probability of surviving both of these critical stages?

5 A card is randomly selected from a deck of 52 cards and placed in a second deck. Then a card is randomly selected from the second deck. What is the probability of getting an ace?

6 If events A and B are independent, prove that the events A and B' are also independent

7 Let $P(A) = 0.3$, $P(B \cup C) = 0.4$, and $P[(A \cap B) \cup (A \cap C)] = 0.2$. Find $P(A \cup B \cup C)$. Use any method you want.

- 8 Bean seed from supplier A have a 90% germination rate and those from supplier B have an 80% germination rate. A seed packing company purchases 45% of their bean seeds from supplier A and 55% from supplier B and mixes these seeds together.
- (a) Find the probability that a seed selected at random from the mixed seeds will germinate, say $P(G)$.
 - (b) Given that a seed does **not** germinate, find the probability that the seed was purchased from supplier B.
- 9 If $P(A) = 0.8$, $P(B) = p$, and $P(A \cup B) = 0.9$, find the value of p if
- (a) A and B are independent.
 - (b) A and B are mutually exclusive.
 - (c) A is a sub set of B .

10 If $S = A \cup B$, $A \cap B = \phi$, $P(A) = 0.35$, $P(G|A) = 0.75$, and $P(G|B) = 0.85$, find $P(G)$ and $P(B|G)$.

11 A deck of 52 cards contains 13 hearts. Suppose that the cards are shuffled and distributed among four players A, B, C, and D so that each player receives 13 cards.

(a) In how many different ways can A will receive 6 hearts, B will receive 4 hearts, C will receive 2 hearts, and D will receive 1 heart?

(b) In how many different ways can each player receive 13 cards?

(c) What is the probability that A receive will 6 hearts, B will receive 4 hearts, C will receive 2 hearts, and D will receive 1 heart?