Probability and Statistics
Test 2
Name:……………………………………………………

1 Fill in the blanks of the following definition. **(9pts)**

The probability mass function (p.m.f.) of a discrete random variable X is a function that satisfies the following properties:

(a) ________________________________;

(b) ________________________________;

(c) ____ .

2 Find the constant c so that $f(x)$ satisfies the conditions of being a probability mass function of a random variable. **(10pts)**

$f(x) = c\binom{3}{x}\binom{7}{2-x}$, x = 0, 1, 2. Note that $\binom{n}{x} = \frac{n!}{x!(n-x)!}$.

3 Let $f(x) = \frac{x}{6}$, x = 1, 2, 3. Find the following. **(12pts)**

(a) $Var(X)$

(b) $Var(2X+5)$

4 Suppose a basketball player can make a free throw 90% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots. Find $P(X=15)$. **(9pts)**

5 The American Almanac of Jobs and Salaries, reported that 30% of accountants are employed in public accounting. Assume that this percentage applies to a group of 10 college graduates just entering the accounting profession. **(12pts)**

(a) Find the probability that at least 3 graduates will be employed in public accounting.

(b) Find the probability that at most 3 graduates will be employed in public accounting.

(c) Find the probability that less than 3 graduates will be employed in public accounting.

(d) Find the probability that more than 3 graduates will be employed in public accounting.

6 If $f(x) = q^{x-1}p$ for $x = 1, 2, \ldots$, then show that $\sum_{x=1}^{\infty} f(x) = 1$. **(6pts)**

Note that $q = 1 - p$.

7 If $f(x) = q^{x-1}p$ for $x = 1, 2, ...,$ then show that $E(X) = \frac{1}{p}$. **(10pts)**

Note that $q = 1 - p$.

8 If $f(x) = \binom{7}{x}\left(\frac{1}{4}\right)^{x}\left(1-\frac{1}{4}\right)^{7-x}$ for x = 0, 1, 2, … , 7, then **derive** the moment generating function $M_X(t)$. **(10pts)**

9 Fit a Poisson model to the following data. **(10pts)**

X	Observed Frequency	Predicted Freq.
0	10	
1	6	
2	3	
3	1	
4 or more	0	

$\hat{l}$ =____________

11 A certain type of aluminum screen that is two feet wide has on the average one flaw in a 100-foot roll. Find the probability that a 50-foot roll has no flaws. **(6pts)**

12 If $M_X(t) = \exp\left\{7\left[t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right]\right\}$, then **find** the mean of the random variable X by using $M_X(t)$. **(6pts)**

Note that $\exp(Y) = e^Y$.