Probability and statistics
Test 2
Fall 2005
Name:…………………………………….
(4+2+2+4)+(2+2+2+2+2)+(6+2+2)+(6+2+2)+(4+2+2+2+2)+((5+5)+(5+5)+(7+3)+10+6

1 Let $f(x) = c\left(\frac{1}{3}\right)^{x}$, $x = 1, 2, \ldots$ Find the following:

(a) c.

(b) $P(X = 2)$

(c) $P(X = 4)$

(d) Find $P(X \in A)$, where $A = \{2, 4, 6, \ldots\}$ Note that A is an infinite set.

If you can't find the value of c in part (a), find the answers of the other parts in terms of c.

2 Let $f(x) = \frac{1}{4}$ for $x = 1, 2, 3, 4$. Find the following:

(a) $E(X)$

(b) $E[X(X-1)]$

(c) $Var(X)$

(d) $E(2X+3)$

(e) $Var(2X+3)$

3 Let an urn has 4 white balls and 5 black balls. Take three balls one at a time **without** replacement. Let X be the number of white balls drawn. Find the following:

(a) The probability mass function (p.m.f.) of X.

(b) $P(X \geq 1)$.

(c) Mean and variance.

4 Let an urn has 4 white balls and 5 black balls. Take three balls one at a time **with** replacement. Let X be the number of white balls drawn. Find the following:

(a) The probability mass function (p.m.f.) of X.

(b) $P(X \geq 1)$.

(c) Mean and variance.

5 Let $f(x) = \dfrac{e^{-2} 2^x}{x!}$ for $x = 1, 2, \ldots$ Find the following:

(a) $E\left(e^{tx}\right)$

(b) $\dfrac{d}{dt}\left[E\left(e^{tx}\right)\right]$

(c) $\dfrac{d^2}{dt^2}\left[E\left(e^{tx}\right)\right]$

(d) Mean

(e) Variance

6 Let $f(x) = pq^{x-1}$ for $x = 1, 2, \ldots$ Prove the following:

(a) $\sum_{x=1}^{\infty} f(x) = 1$

(b) $E(x) = \dfrac{1}{p}$.

7 Suppose a basketball player can make a free throw 80% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots. Find the following:

(a) Probability mass function of X. i.e. $f(x)$

(b) $P(X \leq 12)$.

8 If X have a Poisson distribution so that $2P(X=2)=2P(X=0)+P(X=1)$, find the following. Note that for the Poisson distribution $\lambda > 0$.

(a) λ

(c) $E(X^2)$.

9 Derive the moment generating function of **one** of the following distributions.

(a) Binomial. (b) Geometric.

10 Let X have a Binomial distribution with $n = 20{,}000$ and $p = 0.00015$. Use Poisson approximation to find $P(X > 1)$.