Probability and Statistics
Test 2
Fall 06
8+12+12+6+10+10+10+10+10+6+6=100
Name:………………………………………………………………………………

1 Consider the experiment of tossing a fair coin twice. Define the random variable X as the number of Heads.

(a) Draw a tree diagram and write down the sample space.

(b) Find the p.m.f. of X.

2 Let $f(x) = c^x$; $x = 1, 2, \ldots$ and $0 < c < 1$.

(a) Find the value of c.

(b) Find the $P(X \text{ is even})$.

(c) Find the $P(A)$, where $A = \{3, 6, 9, \ldots\}$.

3 Let $f(x)=\frac{x}{3}$, $x=1,2$. Find the following:

(a) Mean.

(b) Variance.

(c) $Var(3+2x)$.

(d) $E\left(x+\frac{1}{2x}\right)$.

4 Find the sample mean, variance and standard deviation of the following data.

2 3 7 8

5 Let X have a Binomial distribution with mean 11.2 and variance 2.24.

(a) What is the m.g.f. of X ?

(b) Find $P(X \le 2)$.

6 Derive the m.g.f. of Binomial, or Geometric, or Poisson distribution.

7 A container has five black marbles and one white marble. Randomly select one marble with replacement.

(a) What is the probability that the sixth marble selected is the first white marble.

(b) Suppose you want to repeat until you get a white marble for the third time. Find the probability that you have to try at most 5 times.

8 Let $f(x) = c$, for $x = 1, 2, \ldots, 5$.

(a) Find the value of the constant c.

(b) Find $P\left[(X-4)(X-2) \geq 0\right]$.

9 Prove that for a Geometric distribution, $E(X) = \frac{1}{p}$. Do not use m.g.f.

10 Let X have a Poisson distribution so that $3P(X = 1) = P(X = 2)$. Find $P(4.5 < X < 7.5)$.

11 Let X have a Binomial distribution with n = 2,000 and p =0.0015. Use Poisson approximation to find $P(X > 1)$.