Probability and Statistics

Test 2

Fall 08

Name:……………………………………………………….

5+15+10+10+20+10+10+6+6+6+4 =102

1. Filling the blanks of the definition of a probability mass function
   The p.m.f $f(x)$ of a random variable X is a function that satisfies the following properties:

   a.

   b.

   c.

2. Let $f(x) = \frac{x^2}{2}$ for $x = -1, 1$. Find the following:

   a. $E(X)$

   b. $\frac{1}{E[2X+1]}$

   c. $Var(X)$

   d. $E(5-3X)$

   e. $Var(4-3X)$

3. Suppose a basketball player can make a free throw 90% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots.

   a. Find $P\left(X < 13\right)$.

   b. What is the mean of X ?

4. Let the random variable X have a Moment Generating Function $M_X\left(t\right) = \left(.3 + .7e^t\right)^5$. Find $P\left(X > 4 \mid X > 3\right)$

5. Consider the following experiment. An urn contains 4 black balls and 16 white balls.

a. Let X be the number of black balls in the sample. Find $P(X=2)$ if 3 balls are drawn **with** replacement.

b. Let X be the number of black balls in the sample. Find $P(X=2)$ if 3 balls are drawn **without** replacement.

c. If the balls are drawn **with replacement** and the 1st black ball is drawn at the X^{th} trial, then find $P(X=3)$.

d. If the balls are drawn **with replacement** and the 2nd black ball is drawn at the X^{th} trial, then find the $P(X=3)$.

6. In a lot of 100 light bulbs, there are 3 defective bulbs. An inspector inspects 5 bulbs selected randomly. Find the probability of finding at most 1 defective bulbs.

7. A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be [played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay $1000 for each day, up to two days, that the opening game is postponed. Assume that the insurance company determines the number of consecutive days of rain, X, beginning of April 1 is a Poisson random variable with mean 0.6.
    a. If Y is the amount the insurance company will have to pay, what is the relationship between Y and the number of consecutive days of rain?
    b. What is the average amount the insurance company will have to pay? i.e. $E(Y)$ Set up.

8. In Kansas City before tax cost of a minor fender bender repair, X , has a distribution with mean of \$1,700 and variance of \$250,000.
    a. If there is a 0.08 = 8% tax, what is the variance of the after tax cost?
    b. In addition, if there is a \$500 deduction, what is the average cost to an insurance company per minor fender bender in Kansas City?

9. If X has a Poisson distribution with $P\left(X<2\right)=3P\left(X<1\right)$, then find $P\left(X=4\right)$.

10. Let $M_X(t) = 0.4 + 0.6e^t$.

a. Find $M_X^{(1)}(t)$, $M_X^{(2)}(t)$, and $M_X^{(3)}(t)$.

b. Find $E\left[X(X+1)(X+2)\right]$.

11. If $M_X(t) = \sum_{x=1}^{6} \frac{1}{6} e^{tx}$, what is the distribution and what is $E(X)$?