Probability and Statistics

Test 2 (50 minutes, If Dr. Wade can give you a few more minutes it is up to him)

Fall 2009, October 14

Name:……………………………………………………………………………………………

10+8+16+6+10+10+10+10+4+10+10=104

1. If $f(x) = c\left(\frac{1}{2}\right)^x$, $x = 1, 2, \ldots$, then find the following:
    a. c.
    b. $P(X \geq 4)$.
    c. $P(A)$, where $A = \{5, 7, 9, \ldots\}$.

2. Among 120 applicants for a job only 80 are qualified. If 5 of the applicants selected randomly find the probability distribution of the random variable X where X is the number of qualified candidates among the five selected. I want a well defined $f(x)$.

3. Let $f(x)=\frac{x^2}{10}$ for $x=-2,-1,\ 1,\ 2$. Find the following:

a. $E(X)$.

b. $E\left(X^2\right)$.

c. $E\left(X^3\right)$.

d. $E\left(X^9\right)$.

e. $E\left[(X-1)^3\right]$.

f. $Var(X)$.

g. $Var(2X+1)$.

h. $M_X(t)$.

4. Find the sample mean, variance, and standard deviation of the following set of data: 3, 1, 2, 6, 3.

Also find the Coefficient of variation. $COV=\frac{s}{\overline{X}}$. (Standard deviation divided by the mean)

5. If $E\left[X\left(X+1\right)\right]=6.6$ and $E\left[X\left(X-1\right)\right]=2.6$, then find the mean and variance of the random variable X .

6. Derive the mean of the Binomial distribution. If you use a result or a theorem from your previous mathematics classes, please write the result.

7. Consider the experiment of rolling a six-sided balanced die. Assume the success as getting the side with 6 on the top.
   a. Let the first success is achieved at the X th trial. Find $f(x)$, $P(X > 10)$,and $P(X > 20 \mid X > 10)$.
   b. Let the third success is achieved at the X th trial. Find $f(x)$, $P(X = 10)$,and $P(X = 20 \mid X = 10)$.

8. If X have a Poisson distribution so that $2P(X = 2) = 2P(X = 0) + P(X = 1)$, find $P(X = 3)$.

9. Short writing problem: Consider the following derivation. This is a proof for a concept I discussed in class. I actually gave a different derivation. What does this proof do? Explain. **Extra 4 credits.**

$$\lim_{\substack{n\to\infty \\ p\to 0}} M_X(t)$$

$$= \lim_{\substack{n\to\infty \\ p\to 0}} \left\{ \left(1 - p + pe^t\right)^n \right\}$$

$$= \lim_{n\to\infty} \left\{ \left(1 + \frac{\lambda}{n}\left(e^t - 1\right)\right)^n \right\}$$

$$= e^{\lambda\left(e^t - 1\right)}$$

10. If $M_X(t) = Ae^t + Be^{2t}$, $Var(X) = \frac{2}{9}$, and $A < 0.5$. Use the properties of the MGF and College Algebra to find the mean.