Probability and Statistics
Test 2
Spring 2006
(Little bit too long)
Name:…… (1 point)
15+6+16+6+4+4+6+8+10+8+5+4+8= 99

1 Let $f(x)=\frac{x}{6}$ for $x=1,2,3$. Find the following:

(a) $E(X)$

(b) $E\left(\frac{1}{X}\right)$

(c) $Var(X)$

(d) $E(3X+5)$

(e) $Var(3X+4)$

2 Suppose a basketball player can make a free throw 70% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots. Find $P(X=14)$.

3 Consider the following experiment. An urn contains 4 black balls and six white balls.

(a) Let X be the number of black balls in the sample. Find $P(X=3)$ if three balls are drawn with replacement.

(b) Let X be the number of black balls in the sample. Find $P(X=3)$ if three balls are drawn without replacement.

(c) If the balls are drawn with replacement and the first black ball is drawn at the X^{th} trial, then find $P(X=3)$.

(d) If the balls are drawn with replacement and the second black ball is drawn at the X^{th} trial, then find the $P(X=3)$.

4 In a lot of 50 light bulbs, there are 3 defective bulbs. An inspector inspects 6 bulbs selected randomly. Find the probability of finding at least two defective bulbs.

5 Find the sample mean and the sample variance of the following data:

7 9 3 5

6 Consider the following cumulative Binomial distribution and answer the following questions:

x	$F(x)$
0	0.078
1	0.337
2	0.683
3	0.913
4	0.990
5	1.000

(a) $P(X \leq 0)$

(b) $P(X \leq 3)$

(c) $P(X > 3)$

(d) $P(X = 3)$

7 Let X has a discrete Uniform distribution with $\sigma^2 + 1 = \mu$ (Variance +1 = Mean) and a positive variance. Find the probability mass function of X. You may want to find the value of m.

8 If X has a discrete Uniform distribution with $m = 5$, then plot $F(X)$ and also find $P\left(X^2 - 2X + 1 > 0\right)$.

9 Consider the geometric distribution. Show that $\sum_{x=1}^{\infty} f(x) = 1$ and derive the mean.

10 If $M_X(t) = 0.5e^{-t} + 0.5e^{t}$, then show that $E\left(X^r\right) = 0$ when r is odd and $E\left(X^r\right) = 1$ when r is even and find $Var(Y)$ where $Y = X^2$.

11 Let X has a Binomial distribution with $9\sigma^2 = 3\mu = 24$. What is the moment generating function of X ?

$9(Variance) = 3(mean) = 24$

12 Consider the Poisson distribution with $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0,1,2,\ldots$ If $\frac{F(1)}{F(0)} = 3$, find the mean of the distribution.

13 Let $f(x) = c\left(\frac{1}{2}\right)^x$ for $x = 1,2,....$ and $A = \{1,3,5,...\}$

(a) Find the value of c.

(b) Find $P(A)$.

(c) Which probability is greater, X being an odd number or X being an even number? Explain.