

Probability and Statistics

Test 2:.....

Spring 2007

$$10+8+8+10+15+6+8+10+10+10+5=100$$

1 Let $f(x) = c \left(\frac{1}{2}\right)^x$; $x = 0, 1, 2, \dots$

(a) Find the value of c .

(b) Find $P(X \text{ is an odd number})$. Simplify the answer.

2 Let a container has 4 black balls and 6 while balls. All of them are identical other than the color. Randomly select two balls **without** replacement. Let X be the number of black balls in the sample. Find the probability mass function of X . What is the name of the distribution of X ?

3 Let a container has 4 black balls and 6 while balls. All of them are identical other than the color. Randomly select two balls **with** replacement. Let X be the number of black balls in the sample. Find the probability mass function of X . What is the name of the distribution of X ?

4 Let X have a Binomial distribution with mean 6 and variance 3.6.

- (a) Find n and p .
- (b) Find the $P(X > 8 | X > 3)$.

5 Let $f(x) = \frac{x}{3}$, $x = 1, 2$. Find the following

(a) μ .

(b) σ^2

(c) $E\left(\frac{1}{X(2X+1)}\right)$.

(d) $Var(2+3X)$.

(e) $Var(X^2)$.

6 Find the sample mean and sample variance of the following data.

1, 2, 6

7 Consider the Discrete Uniform distribution with $F(4) - F(3) = 0.2$.
Find $P[(X - 2)(X - 4) \leq 0]$.

8 Let the moment generating function of a distribution, $M_X(t) = Ae^t + Be^{2t}$,
If $E(X) = 1.25$, then find the $Var(X)$.

9 Derive the moment generating function of the Geometric distribution.

- 10 Consider the experiment of rolling a six-sided balanced die until you get the third six. Assume you get the third six on the X th trial.
- (a) What is the probability mass function of X .
 - (b) Find $P(X < 6)$.

- 11 Let $X \sim \text{Bin}(2000, 0.001)$, use the Poisson approximation to find $P(X \leq 2)$.