Probability and Statistics
Test 2
Spring 2009
Name:……………………………………………………………………
15+12+12+10+5+10+10+6+10+10=100

1 Let $f(x)=\frac{2x+1}{8}$ for $x=1,2$. Let $Y=X^2$. Find the following:

a. $E(X)$

b. $E\left(X^2\right)$

c. $E\left(X^4\right)$

d. $Var\left(X^2\right)$

e. $Var(3X+4)$

2 Let X equal the number of rolls of a balanced six-sided die that are required to observe the **second** six on the top.

a. Find the p.m.f. of X.
b. Give the values of the mean, variance, and standard deviation of X.
c. Find $P(X>4)$.

3 Consider the following experiment. An urn contains 4 black balls and six white balls.

a. Let X be the number of black balls in the sample. Find $P(X = 3)$ if three balls are drawn with replacement.

b. Let X be the number of black balls in the sample. Find $P(X = 3)$ if three balls are drawn without replacement.

c. If the balls are drawn with replacement and the first black ball is drawn at the X^{th} trial, then find $P(X = 3)$.

d. If the balls are drawn with replacement and the second black ball is drawn at the X^{th} trial, then find the $P(X = 3)$.

4 In a lot of 50 light bulbs, there are 3 defective bulbs. An inspector inspects 6 bulbs selected randomly. Find the probability of finding at least two defective bulbs.

5 Consider the geometric distribution. Show that $\sum_{x=1}^{\infty} f(x) = p + qp + qp^2 + ... = 1$.

6 Let $f(x) = c\left(\frac{1}{2}\right)^x$ for $x = 2, 3, 4, ...$ and $A = \{3, 5, 7, ...\}$

a. Find the value of c.

b. Find $P(A)$.

7 For a Poisson distribution, show that $f(x+1)=\frac{\lambda}{x+1}f(x)$ for $x=0, 1, 2\dots$. If $f(0)=e^{-2}$, then find $f(1)$ and $f(2)$ using $f(x+1)=\frac{\lambda}{x+1}f(x)$. (Hint: Pick the correct x values. You need to find the value of λ too.)

8 If X have a Poisson distribution so that $2P(X=2)=2P(X=0)+P(X=1)$, find λ. Note that for the Poisson distribution $\lambda>0$.

9 Derive the moment generating function of **one** of the following distributions.

(a) Binomial. (b) Geometric. (c) Poisson.

10 If $f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ for x =0, 1, 2, ….., then the m.g.f. of X is given by $M_X(t) = e^{\lambda(e^t - 1)}$.

a. Find $M_X(1)$ and $M_X(2)$

b. Find $E(e^{2X} + 2e^{X} + 1)$.