1. Let \( f(x) = \frac{2x+1}{8} \) for \( x = 1, 2 \). Let \( Y = X^2 \). Find the following:
   a. \( E(X) \)
   b. \( E(X^2) \)
   c. \( E(X^4) \)
   d. \( Var(X^2) \)
   e. \( Var(3X+4) \)

2. Let \( X \) equal the number of rolls of a balanced six-sided die that are required to observe the second six on the top.
   a. Find the p.m.f. of \( X \).
   b. Give the values of the mean, variance, and standard deviation of \( X \).
   c. Find \( P(X > 4) \).
3  Consider the following experiment. An urn contains 4 black balls and six white balls.
   a. Let $X$ be the number of black balls in the sample. Find $P(X = 3)$ if three balls are drawn with replacement.

   b. Let $X$ be the number of black balls in the sample. Find $P(X = 3)$ if three balls are drawn without replacement.

   c. If the balls are drawn with replacement and the first black ball is drawn at the $X^{th}$ trial, then find $P(X = 3)$.

   d. If the balls are drawn with replacement and the second black ball is drawn at the $X^{th}$ trial, then find the $P(X = 3)$.

4  In a lot of 50 light bulbs, there are 3 defective bulbs. An inspector inspects 6 bulbs selected randomly. Find the probability of finding at least two defective bulbs.
5 Consider the geometric distribution. Show that \[ \sum_{x=1}^{\infty} f(x) = p + qp + qp^2 + \ldots = 1. \]

6 Let \( f(x) = c \left( \frac{1}{2} \right)^x \) for \( x = 2, 3, 4, \ldots \) and \( A = \{3, 5, 7, \ldots\} \)

   a. Find the value of \( c \).
   
   b. Find \( P(A) \).
7  For a Poisson distribution, show that $f(x+1) = \frac{\lambda}{x+1} f(x)$ for $x = 0, 1, 2, \ldots$. If $f(0) = e^{-\lambda}$, then find $f(1)$ and $f(2)$ using $f(x+1) = \frac{\lambda}{x+1} f(x)$. (Hint: Pick the correct $x$ values. You need to find the value of $\lambda$ too.)

8  If $X$ have a Poisson distribution so that $2P(X = 2) = 2P(X = 0) + P(X = 1)$, find $\lambda$. Note that for the Poisson distribution $\lambda > 0$. 
Derive the moment generating function of one of the following distributions.

(a) Binomial.  (b) Geometric.  (c) Poisson.

If \( f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \) for \( x = 0, 1, 2, \ldots \), then the m.g.f. of \( X \) is given by \( M_X(t) = e^{\lambda(e^t - 1)}. \)

a. Find \( M_X(1) \) and \( M_X(2) \)

b. Find \( E\left(e^{2x} + 2e^x + 1\right)\).