Probability and Statistics

Test 2

March 26, 2010

Name:……

10+8+10+12+10+10+10+12+8+(2+2+6)=100

1. The probability mass function (p.m.f.) of a discrete random variable X is a function that satisfies the following properties:

   a. _

   b. _

   c. _

2. Find the value of the constant c if $f(x)=\frac{c}{(x+1)(x+3)}$ for x = 0, 1, 2, 3,…..

3. If $f(x) = c(0.4)^x$ for x=0, 1, 2, ….. Find the value of c and $P(X < 4 | X > 0)$.

4. An urn contains 2 black balls and 3 white balls identical other than the color. Select two balls at random without replacement one at a time. Let X be the number of white balls in the sample. What are the possible values of X ? Draw a tree diagram of the experiment results. Find the p.m.f. of X . What is the name of the distribution of X ?

5. Let $f(x) = \frac{|x|+1}{5}$ for $x = -1, 0, 1$. Find the followings:

   a. $E(X)$

   b. $Var(X)$

   c. $E\{X(X-1)+X\} - [E(X)]^2$

   d. $Var(2X+1)$

   e. $Var(X^2+1)$

6. Show that the variance of the following distribution isof the form $Var(X) = Kp(1-p)$. Find the value of K. What is the distribution of X if a=0 and b=1?

X	a	b
$f(x)$	$1-p$	p

7. Consider the experiment of tossing fair coin till you get two heads. Write down the sample space up to 4 trials. If the second success is at the X th trial, find the distribution of X. What is the probability that X equals five?

8. Answer all the following parts:
    a. Derive the moment generating function of the Geometric distribution.
    b. Take the first and second derivatives of the moment generating function
    c. Derive the mean and variance using parts (a) and (b).

9. In a certain age group of an insurance company's clients, number of accidents per year has a Poisson distribution with mean 0.25. At the end of the year company send a check for $ 100 for those who did not have any accidents, $50 check for those who had one accident and no refunds for those who had 2 or more accidents. What is the expected cost of refunds to the company per client in this age group?

10. Answer the following questions:
    a. What is the approximation for Binomial(20000, 0.0005)

    b. If $X \sim Binomial(15, p)$, what is $F(10) + f(11)$ equal to? Answer should be in terms of $F(\ )$ or $f(\ )$.

    c. Find the sample mean and sample variance of the data 2, 5, 11.