Probability and Statistics
Spring 2005
Test 2
03/02/05
Name:……

6+ 8+ 5+ 6+ 15+ 10+10+10+10+5+5+ 5+5

1 Fill in the blanks of the following definition

The probability mass function $f(x)$ of a discrete random variable X is a function that satisfies the following properties:

(a)

(b)

(c)

2 Find the value of the constant c and $P(A)$, where $A = \{1, 2\}$.

$f(x) = c\left(\frac{1}{3}\right)^x$ for $x = 1, 2, \ldots$. (**This problem has two parts)**

3 In a lot of 50 light bulbs, there are 3 defective bulbs. An inspector inspects 6 bulbs selected randomly. Find the probability of finding at least two defective bulbs. **(Setup do not simplify)**

4 Suppose a basketball player can make a free throw 70% of the time. Let X equals the minimum number of free throws that this player must attempt to make a total of 10 shots. Find $P(X = 14)$.

5 Let $f(x) = \frac{(|x|+1)^2}{9}$ for x = -1, 0, 1

Fill in the blanks and find the following:

X			
$f(x)$			

(a) $E(X)$

(b) $E(X^2)$

(c) $Var(X)$

(d) $E(3X+5)$

(e) $Var(3X+5)$

6 Consider a Binomial distribution with mean 2.8 and variance 2.24. Find the following.

(a) n and p.

(b) $P(X=2)$

(c) $P(X<2)$

(d) $P(X \le 2)$

(e) $P(X>2)$

7 Derive the moment generating function of **one** of the following distributions.

(a) Binomial. (b) Geometric. (c) Poisson.

8 (a) If $M_X(t) = \dfrac{0.4e^t}{1-0.6e^t}$, $t < -\ln(0.6)$, then find $P(1 \le X \le 2)$.

(b) If $M_X(t) = \left(\dfrac{0.2e^t}{1-0.8e^t}\right)^2$, $t < -\ln(0.8)$, then find $P(1 \le X \le 2)$.

9 Consider the Geometric distribution.

(a) Derive the mean of the distribution.

(b) Show that $P(X > n) = q^n$.

10 Let X have a Poisson distribution so that $5P(X=1)=P(X=2)$. Find $P(X=0)$

11 Let X have a Binomial distribution with n=20,000 and p=0.0003. Use Poisson approximation to find $P(X>1)$.

12 Find the **variance** of the Poisson distribution using $M_X(t)$ or a function of it and taking derivatives.

$M_X(t) = e^{\lambda(e^t - 1)}$

13 A bowl contains 2 white balls and 3 black balls. Two balls are drawn **with replacement**. Let X be the number of white balls drawn. Find the distribution of X.

X	
$f(x)$	