

Probability and Statistics

Test 3

Fall 05

Name:.....

1 Let  $f(x) = 2e^{-2x}$  for  $x \geq 0$ . Find the following:

(a) Mean.

(b) Variance.

(c)  $\pi_{0.25}$ . Note  $P(x < \pi_{.25}) = 0.25$ .

(d)  $M_X(t)$ .

2 Let  $f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 \leq x < 1 \end{cases}$ . Find the cumulative distribution function  $F(x)$ .

3 If  $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ , for  $x \geq 0$ , then show that  $P(X > x) = e^{-\frac{x}{\theta}}$ .

4 If  $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ , for  $x \geq 0$ , then prove that  $P(X > x+y | X > x) = P(X > y)$ .

- 5 Let  $f(x) = 6x(1-x)$  for  $0 < x < 1$ . Find the probability density function of  $Y = X^3$ .

- 6 Let  $f(x) = \frac{1}{2}$  for  $-1 < x < 1$  and  $Y = |X| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$ .

Find the probability density function of  $Y$ , which is  $g(y)$ .

7 Let  $f(x) = \frac{1}{2}$  for  $-1 < x < 1$ . Find the probability density function of  $Y = X^2$ .

8 Let  $Z \sim N(0,1)$ . Find the following:

(a)  $P(0 < Z < 2.35)$

(b)  $P(|Z| > 2.35)$

9 Let  $X \sim N(100, 5^2)$ . Find the following:

(a) The values of  $a$  and  $b$   $aX + b \sim N(0, 1)$ .

(b)  $P(X > 110)$

(c) The value of  $c$  if  $P(|X - 100| > c) = 0.0668$ .

(d)  $M_X(t)$ . (Do not have to derive this. Find it anyway you can.)

10 Let  $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x+1}{2} & \text{if } -1 \leq x < 1. \\ 1 & \text{if } x \geq 1 \end{cases}$ .

Find the probability density function of  $X$ ,  $f(x)$ .

11 If  $E(X+4) = 10$  and  $E[(X+4)^2] = 116$ , then find the following:

(a)  $E(X)$ .

(b)  $Var(X)$ .

12 If  $Z \sim N(0,1)$ , then show that  $Z^2 \sim \chi^2_{(1)}$

13 Let  $X \sim \chi^2_{(17)}$ . Find the following:

(a)  $\chi^2_{0.025}$ .

(b)  $\chi^2_{0.975}$

(c) Mean

(d) Variance