Probability and Statistics

Test 3

Fall 08

Name:………………………………………………………………………………………..

(5+5+5)+5+10+(5+5)+10+15+10+5+20=100

1. Let $f(x) = 0.5x^2 e^{-x}$; $x > 0$. Derive the following:

   a. Mean

   b. Variance

   c. Moment generating function.

2. Find the value of the constant c when $f(x) = \dfrac{c}{\sqrt{x}}$; $0 < x < 4$.

3. Find $F(x)$ if $f(x) = |x|$ for $-1 < x < 1$, zero otherwise.

4. Let $f(x) = 2e^{-2x}; x > 0$. Find the following:

    a. Median

    b. $P(X > 0.5)$.

5. Let $f(x) = \dfrac{2^x e^{-2}}{x!}; x = 0,1,2,3,......$ and $Y = \sqrt{X}$. Find the probability mass function of Y.

6. Let $f(x) = \dfrac{5x^4}{33}$; $-1 < x < 2$ and $Y = X^2$. Find the probability density function of Y.

7. Let x_1 and x_2 be two random observations from $f(x) = \dfrac{4}{x^5}$; $x > 1$. Find $P(X_1 > 2 \text{ and } X_2 < 2)$.

8. Let the m.g.f. of X_1 and X_2 be $M_{X_1}(t) = \left(.3 + .7e^t\right)^5$ and $M_{X_2}(t) = \left(.3 + .7e^t\right)^7$ respectively. Also assume that x_1 and x_2 are independent. Find the m.g.f. of $Y = X_1 + X_2$.

9. Let X_1 and X_2 be two independent random variables with respective means 10 and 20 and respective variances 9 and 16. Find the followings;

   a. $E(X_1 + X_2)$
   b. $Var(X_1 + 2X_2)$
   c. $E(X_1 X_2)$
   d. $Var(X_1 X_2)$