

Probability and Statistics

Test 3

Fall 2009

Name:.....

Problems are almost equally weighted.

1 Find the constant c if $f(x) = \frac{c}{1+x^2}$ for $-\infty < x < \infty$.

2 Let $f(x) = xe^x$, $0 < x < 1$ be the p.d.f. of X . Notice the limits of X . Find the mean of the distribution.

3 Let $f(x) = xe^x$, $0 < x < 1$ be the p.d.f. of X . Notice the limits of X . Find the variance of the distribution.

4 Let the p.d.f. of X be $f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 \leq x < 1 \end{cases}$. Find the c.d.f. of X .

5 Derive the moment generating function of X if the p.d.f. of X is $f(x) = 2e^{-2x}$ for $x \geq 0$.

6 Find $P(X > 1 | X > 0.5)$ if X has the p.d.f. of $f(x) = 2e^{-2x}$ for $x \geq 0$.

7 Let X_1 and X_2 be a random sample from an exponential distribution with the notation in the book. Also assume that X_1 and X_2 are independent. Derive the m.g.f. of $Y = X_1 + X_2$. Also **find** the mean.

8 Let X_1 and X_2 be two independent random variables with respective means 1 and 2 and respective variances 4 and 9. Find the followings:

a. $E(X_1 + X_2)$

b. $Var(X_1 + 2X_2)$

c. $E(X_1X_2)$

d. $Var(X_1X_2)$

9 Let X has a continuous uniform distribution with $a = 6$ and $b = b, b > a$. If $E(X) = 6 Var(X)$, then find $P\left(X + \frac{35}{X} > 12\right)$. Use cover page for formulas.

10 Find the p.d.f. of $Y = X^2$ if $f(x) = \frac{1}{3}$, $-2 < x < 1$.

11 Let the p.d.f. of X be $f(x) = 4x^3$ for $0 < x < 1$. Find the p.d.f. of $Y = X^4$ and then $P(Y > 0.7)$.

12 Derive the median of X if the p.d.f. of X is $f(x) = 2e^{-2x}$ for $x \geq 0$.