Probability and Statistics
Spring 2005
Test 3

Name:…….

15+10+10+15+15+10+5+10+5+5=100

1 Let $f(x)=\frac{3x^2}{54}$, $-c<x<c$. Find the following;

(a) c.

(b) $F(x)$.

(c) $P(-1<X<1)$.

2 Let $f(x) = 2e^{-2x}$, $x > 0$. **Find** $E(X)$ **and** $E\left(X^2\right)$**.**

3 Let $f(x) = 2e^{-2x}$, $x > 0$. Find the following;

(a) Median.

(b) $P(X > 2 + median \mid X > median)$.

4 Let $f(x)=\frac{1}{3}e^{-\frac{x}{3}},\ x>0$.

(a) Derive the moment generating function (m.g.f.) and show that it is $M_X(t)=\frac{1}{(1-3t)},\ t<\frac{1}{3}$.

(b) Find the mean by **taking the derivatives** of the m.g.f.

5 Let $Z \sim N(0,\ 1)$. Find the following:

(a) $P(Z < 2.35)$.

(b) $P(|Z| > 2.35)$.

(c) $Z_{0.0228}$.

(d) a constant c such that $P(|Z| < c) = 0.9544$.

6 Let $M_X(t) = e^{10t+18t^2}$. Find the following:

(a) $P(X > 23.8)$.

(b) A constant c such that $P(|X - 10| < c) = 0.9544$.

7 Let the probability density function of X be $f(x) = \frac{1}{16}x^2e^{-x/2}$, $x \geq 0$. Find the probability density function of $Y = \sqrt{X}$.

8 Find the p.d.f. of $Y = X^2$ if $f(x) = \frac{1}{4}$, $-1 < x < 3$.

9 Let the p.d.f. of X be given by the following table.

X	-1	0	1	2
$f(x)$	0.25	0.25	0.25	0.25

Find the p.d.f of $Y = X^2$.

10 Let $F(x) = \begin{cases} 0 \text{ if } x < 0 \\ x^2 \text{if } 0 \le x < 1. \\ 1 \text{ if } x \ge 1 \end{cases}$

Find $f(x)$.