

Probability and Statistics
Test 3, Part 1

Name:.....

$$10+6+6+2+6+6+10+8+8+8=70$$

1 Let $f(x) = c(x-1)$; $1 \leq x \leq 1.5$. Find the following:

(a) c .

(b) $F(x)$.

(c) $F(1.25)$.

(d) $P(X > 1.25)$.

(e) $P(X \geq \text{median})$.

2 Let $F(x) = \begin{cases} 1 - e^{-x^2}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$. Find $f(x)$.

3 Let $f(x) = \frac{1}{5}$, $0 < x < 5$. Find $P(X > 3 | X > 1)$.

4 Let $f(x) = 2e^{-2x}$, $x \geq 0$. Find $P(X = 2)$.

5 Let $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$. If $P(X > 5) = e^{-20}$, find the value of λ .

6 Let $f(x) = 2e^{-2x}$, $x \geq 0$. Find $P(X > 6 | X > 4)$.

7 Let the probability density function (p.d.f.) of X be $f(x) = \begin{cases} 0.5, & 0 < x < 1 \\ 1.0, & 1 \leq x < 1.5 \end{cases}$.

- (a) Find the cumulative distribution function (c.d.f.).
- (b) Find $E(X)$.

8 **Derive** the m.g.f. of $f(x) = xe^{-x}$, $x \geq 0$.

- 9 Let $f(x) = \frac{1}{2}$, $-1 < x < 1$, zero otherwise. Consider the transformation $Y = X^2$
Find the p.d.f. of Y .

- 10 If $M_X(t) = \frac{1}{(1-2t)^5}$, then find the mean and variance of X .

Probability and Statistics

Test 3, Part 2

$$6+6+6+6+6=30$$

- 11 Let X_1 and X_2 be two independent Poisson random variables with respective variances 2 and 3. Find $P(X_1 + X_2 = 2)$.

- 12 Let X_1 and X_2 be a random sample of size 2 from $f(x) = \frac{1}{2}e^{-\frac{x}{2}}$, $x \geq 0$. Find $P(X_1 > 2, X_2 < 3)$.

- 13 Let X_1 and X_2 be a random sample of size 2 from $f(x) = \frac{1}{2}e^{-\frac{x}{2}}$, $x \geq 0$. Find $E[X_1^3(X_2 - 2)]$

- 14 Let X_1 and X_2 be a random sample of size 2 from $f(x) = 6x(1-x)$, $0 < x < 1$. Find the mean and variance of $Y = 2X_1 + 3X_2$.

- 15 Let X_1 , X_2 , and X_3 be mutually independent random variables with Poisson distributions having means 2, 1, 4, respectively. Find the moment generating function of $Y = X_1 + X_2 + X_3$.