

Probability and Statistics
Test 3
Spring 2007

Name:.....

6+6+6+8+10+10+(3+2+3)+10+10+(6+2)+10+8=100

There are no errors in the problems. Read them carefully.

1 Let $f(x) = \frac{c}{\sqrt{x}}$, $0 < x < 1$. Find the value of c .

2 Let $f(x) = cx^2e^{-x^3}$, $0 < x < \infty$. Find the value of c .

3 Let $F(x) = \begin{cases} c(1 - e^{-x^2}) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$, Find the value of c .

4 Let $f(x) = \frac{200}{x^3}$, $x > 10$. Find the median ($\pi_{0.5}$) of the distribution.

5 Let $f(x) = 0.5x^2e^{-x}$, $0 < x < \infty$. Find the mean and variance.

6 Let $f(x) = \frac{|x|}{10}$, $-2 < x < 4$. Let $Y = X^2$. Find the p.d.f. of Y .

7 Let $f(x) = \frac{1}{250} e^{-\frac{x}{250}}$, $0 < x < \infty$. Find the following:

- (a) $P(X > 173)$.
- (b) $P(X > 250)$.
- (c) $P(X > 173 | 250)$.

8 Let $f(x) = \lambda e^{-\lambda x}$, $0 < x < \infty$. Derive the moment generating function using the definition.

9 Let $f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$. Find the c.d.f. of X , $F(x)$.

10 Let X_1 and X_2 be two independent random variables with probability density functions $f_1(x_1) = 2x_1$, $0 < x_1 < 1$ and $f_2(x_2) = 3x_2^2$, $0 < x_2 < 1$. Find the following:

(a) $P(0 < X_1 < 0.5, 0.5 < X_2 < 1)$.

(b) $P(\pi_{0.3} < X_1 < \text{median}, \text{median} < X_2 < 1)$.

11 Let X_1 , X_2 , and X_3 be a random sample of size three from $f(x) = 6x(1-x)$ for $0 < x < 1$. Find the mean and variance of $Y = 3X_1 + 2X_2 + X_3$.

12 Let $X_1 \sim Poi(2)$, $X_2 \sim Poi(3)$, and $X_3 \sim Poi(7)$. Assume that X_1 , X_2 , and X_3 are mutually independent. Find the distribution of $Y = X_1 + X_2 + X_3$.