

Probability and Statistics
Test 3
Spring 2009

Name:.....

$$7+7+8+10+12+10+10+10+8+10+8=100$$

There are no errors in the problems. Read them carefully.

1 Let $f(x) = c(9 - x^2)$, $-3 < x < 3$. Find the value of c .

2 Let the p.d.f. of X be $f(x) = \frac{x}{2}$, $0 < x < 2$. Find the p.d.f. of $Y = 4 - X^3$.

3 Let $f(x) = \frac{200}{x^3}$, $x > 10$. Find the first quartile ($\pi_{0.25}$) of the distribution.

4 Let $f(x) = 0.5x^2e^{-x}$, $0 < x < \infty$. Let $Y = X^2$. Find $Var(Y)$.

5 $f(x) = \frac{|2x|}{5}$, $-1 < x < 2$. Let $Y = X^2$. Find the p.d.f. of Y .

6 Let $f(x) = \frac{1}{25} e^{-\frac{x}{25}}$, $0 < x < \infty$. Find the following:

- a. $P(X > 17)$.
- b. $P(X > 25)$.
- c. $P(X > 17 | X > 25)$.

7 Let $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $0 < x < \infty$. Derive the moment generating function using the definition.

8 Let the p.d.f. of X be $f(x) = \begin{cases} 0.1; & 0 < x < 1 \\ 0.2; & 1 \leq x < 2 \\ 0.3; & 2 \leq x < 3. \\ 0.4; & 3 \leq x < 4 \\ \text{otherwise} \end{cases}$. Find the c.d.f. of X , $F(x)$.

- 9 Let X_1 and X_2 be two independent random variables with probability density functions $f_1(x_1) = 2x_1$, $0 < x_1 < 1$ and $f_2(x_2) = 3x_2^2$, $0 < x_2 < 1$. Find the following:
- $P(0 < X_1 < 0.5, 0.5 < X_2 < 1)$.
 - $P(\pi_{0.3} < X_1 < \text{median}, \text{median} < X_2 < 1)$.

10 Let X_1 , X_2 , and X_3 be a random sample of size three from $f(x) = 6x(1-x)$ for $0 < x < 1$. Find the mean and variance of $Y = 3X_1 + 2X_2 + X_3$.

11 Let $X_1 \sim Poi(2)$, $X_2 \sim Poi(3)$, and $X_3 \sim Poi(7)$. Assume that X_1 , X_2 , and X_3 are mutually independent. Find the distribution of $Y = X_1 + X_2 + X_3$.