

Probability and Statistics

Test3, Spring 2010

Name:.....

$$5+10+5+12+8+6+12+10+12+10+10=100$$

1. If $F(x) = 1 - e^{-\left(\frac{x}{3}\right)^2}$ for $x > 0$, then find $f(x)$.

2. If $f(x) = |x|$ for $|x| < 1$, then find $F(x)$

3. If $f(x) = cx^2 e^{-3x}$ for $x > 0$, then find the value of C .

4. If $f(x) = 2e^{-2x}$ for $x > 0$, then derive (not find) the followings:

- a. Mean
- b. Variance
- c. Median

5. Let $f(x) = xe^{-x}$ for $x > 0$. Find $M_X(t)$.

6. Let $f(x) = \frac{|x|+2}{8}$ for $x = -1, 0, 1$. Find the p.m.f. of $Y = X^2$.

7. Let $f(x) = \frac{|x|}{10}$, $-2 < x < 4$. Let $Y = X^2$. Find the p.d.f. of Y .

8. Let $X_1 \sim Poi(5)$, $X_2 \sim Poi(7)$, and X_1 and X_2 are independent. Find $P(X_1 + X_2 = 2)$.

9. Let X_1 and X_2 be independent random variable with probability density functions $f_1(x_1) = 3x_1^2$

for $0 < x_1 < 1$ and $f_2(x_2) = \frac{1}{4}$ for $0 < x_2 < 4$. Find the following:

a. $P(x_1 < .5, x_2 > 2)$

b. $Var(X_1X_2)$.

10. Let X_1 and X_2 be independent random variable with moment generating functions

$M_{X_1}(t) = e^{10t+2t^2}$ and $M_{X_2}(t) = e^{50t+8t^2}$. Find the following if $Y = X_1 + X_2$:

a. $M_Y(t)$

b. $E(Y)$

11. Let $X_1, X_2,$ and X_3 be a random sample of size three from $f(x) = 6x(1-x)$ for $0 < x < 1$. Find the mean and variance of $Y = 3X_1 + 2X_2 + X_3$.