Let $X_1$, $X_2$, and $X_3$ be a random sample from a Bernoulli distribution with $P(X = 0) = 0.4$. Find $P(X_1 + X_2 + X_3 \leq 1)$.

Let $X_1$ and $X_2$ be a random sample of size 2 from the exponential distribution with $f(x) = e^{-x}$ for $x \geq 0$. Find the value of $P(\max(X_1, X_2) < 2) = P(X_1 < 2 \text{ and } X_2 < 2)$.

Let $X_1$ and $X_2$ be a random sample of size 2 from exponential distribution with parameter $\theta$. Find the moment generating function of $Y = X_1 + X_2$. Recognize the distribution of $Y$ using the m.g.f. Also report it.
4 Let $X_1$ and $X_2$ be two independent random variables with respective means $\mu_1$ and $\mu_2$ and variances $\sigma_1^2$ and $\sigma_2^2$. Prove the following by first principles.

(a) $E[(X_1 - \mu_1)(X_2 - \mu_2)] = 0$

(b) $Var(X_1 + X_2) = \sigma_1^2 + \sigma_2^2$.

5 Let $X_1 \sim N(10, 3^2)$, $X_2 \sim N(20, 4^2)$, and $X_1$ and $X_2$ are independent. Find the moment generating function of $Y = X_1 + X_2$. Also find $P(Y > 40)$. 
6 Let $X_1 \sim N(0, 2^2)$ and $X_2 \sim N(0, 3^2)$. Assume $X_1$ and $X_2$ are independent. Find the following:

(a) $P \left[ \frac{X_1^2}{4} + \frac{X_2^2}{9} > 5.991 \right]$. \\
(b) $P(X_1^2 > 20.096)$. \\

7 Let $X_1$, $X_2$, and $X_3$ be three independent random variables with respective means 1, 2, and 3, and variances 4, 9, and 16. Find the following:

(a) $E(X_1^2)$. \\
(b) $Var(2X_1)$. \\
(c) $E(X_1X_2X_3)$. \\
(d) $Var(X_1X_2X_3)$. \\
8 Let $X_1, X_2, \ldots, X_{16}$ be a random sample from $N(40, 6^2)$. What is the distribution of $Y = \sum_{i=1}^{16} X_i$. Find the constant $c$ such that $P(Y \leq c) = 0.9772$.

9 Let $X$ be a random variable with mean 50 and standard deviation 14. Let $\bar{X}$ be the sample mean of a random sample of size 49 from this distribution. Find $P(48 \leq \bar{X} \leq 54)$. 