Probability and Statistics
Test 4
Fall 2005

Name:………………………………………………………..

1 Let X_1, X_2, and X_3 be a random sample from a Bernoulli distribution with $P(X = 0) = 0.4$. Find $P(X_1 + X_2 + X_3 \leq 1)$.

2 Let X_1 and X_2 be a random sample of size 2 form the exponential distribution with $f(x) = e^{-x}$ for $x \geq 0$. Find the value of $P(\max(X_1, X_2) < 2) = P(X_1 < 2 \text{ and } X_2 < 2)$.

3 Let X_1 and X_2 be a random sample of size 2 from exponential distribution with parameter θ. Find the moment generating function of $Y = X_1 + X_2$. Recognize the distribution of Y using the m.g.f. Also report it.

4 Let X_1 and X_2 be two independent random variables with respective means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. Prove the following by first principles.

(a) $E\left[\left(X_1 - \mu_1\right)\left(X_2 - \mu_2\right)\right] = 0$

(b) $Var\left(X_1 + X_2\right) = \sigma_1^2 + \sigma_2^2$.

5 Let $X_1 \sim N\left(10, 3^2\right)$, $X_2 \sim N\left(20, 4^2\right)$, and X_1 and X_2 are independent. Find the moment generating function of $Y = X_1 + X_2$. Also find $P\left(Y > 40\right)$.

6 Let $X_1 \sim N\left(0, 2^2\right)$ and $X_2 \sim N\left(0, 3^2\right)$. Assume X_1 and X_2 are independent. Find the following:

(a) $P\left[\frac{X_1^2}{4} + \frac{X_2^2}{9} > 5.991\right]$.

(b) $P\left(X_1^2 > 20.096\right)$.

7 Let X_1, X_2, and X_3 be three independent random variables with respective means 1, 2, and 3, and variances 4, 9, and 16. Find the following:

(a) $E\left(X_1^2\right)$.

(b) $Var(2X_1)$.

(c) $E(X_1 X_2 X_3)$.

(d) $Var(X_1 X_2 X_3)$.

8 Let $X_1, X_2, \ldots, X_{16}$ be a random sample from $N\left(40, 6^2\right)$. What is the distribution of $Y = \sum_{i=1}^{16} X_i$. Find the constant c such that $P\left(Y \le c\right) = 0.9772$.

9 Let X be a random variable with mean 50 and standard deviation 14. Let $\bar{X}$ be the sample mean of a random sample of size 49 from this distribution. Find $P\left(48 \le \bar{X} \le 54\right)$.