

Probability and Statistics, Test 4, Fall 08

Name:.....

$$15+15+15+10+10+15+10+10=100$$

1. If  $Z \sim N(0,1)$ , then find the following:

a.  $P(|Z| > 2.35)$ .

b.  $P(|Z-1| < 0.5)$ .

c.  $P(Z^2 > 3.8416)$

2. If  $X \sim N(250, 225)$  then find the followings:

a.  $P(X > 275)$

b.  $P(|X - 250| > 30)$

c. Constant  $c$  if  $P(|X - 250| > c) = 0.0718$

3. Let  $X_1$  and  $X_2$  be two independent random variables with respective moment generating functions (m.g.f.)  $M_{X_1}(t) = e^{10t+18t^2}$  and  $M_{X_2}(t) = e^{20t+8t^2}$ .

a. Derive the m.g.f. of  $Y = 2X_1 + X_2$ .

b. Find  $P(Y > 64.8)$ .

4. Let  $X_1, X_2, \dots, X_{25}$  be a random sample of size 25 from  $N(100, 400)$ . Find  $P(\bar{X} > 110)$ .

5. Let  $X_1, X_2, \dots, X_{16}$  be a random sample from  $N(46, 6^2)$ . Find  $P\left(688 \leq \sum_{i=1}^{16} X_i \leq 760\right)$ .

6. Let  $X$  be a random variable with mean 100 and variance 196. Let  $\bar{X}$  be the sample mean of a random sample of size 49.

- a. What is the approximate distribution of  $\bar{X}$ ? (name of the distribution, mean, variance)
- b. What result did you use here?
- c. Find  $P(\bar{X} > 103)$ .

7. Let  $X_1, X_2, \dots, X_{48}$  be a random sample from a distribution with p.d.f  $f(x) = 1, 0 < x < 1$ . Find  $P(\bar{X} > 0.6)$ .

8. If  $X \sim N(\mu, \sigma^2)$ , then prove that  $U = \left(\frac{X - \mu}{\sigma}\right)^2 \sim \chi_{(1)}^2$ .