Probability and Statistics

Test 4

Fall 2009

Name:…………………………………………………………………………………………………………..

16+14+14+14+14+14+14+14=100

1. Let $Z \sim N(0,1)$. Find the following:
   a. $P(Z > -2.38)$
   b. $P(|Z| > 2.38)$
   c. Constant $c$ such that $P(Z^2 > c) = 0.05$
   d. $P(2Z + 5 > 0.24)$

2. Let $X \sim N(10,16)$. Find the following:
   a. $P(X > 20.4)$
   b. $P\left[(X-10)^2 > 61.456\right]$

3. If $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}$ ; $-\infty < z < \infty$, then derive the distribution of $Y = Z^2$.

4. If $X_1 \sim N(100, 2^2)$ and $X_2 \sim N(80, 3^2)$, then find the following:

   a. The distribution of $Y = 2X_1 + X_2$

   b. $P(Y > 290)$.

5. Let $X_1$, $X_2$, and $X_3$ be $N(60, 3^2)$, $N(48, 4^2)$, and $N(51, 2^2)$, respectively. Find $P\left(\dfrac{X_1 + X_2}{2} > X_3\right)$. Assume $X_1$, $X_2$, and $X_3$ are mutually independent.

6. Let $X_1$, $X_2$,..., $X_{16}$ be a random sample from $N(40, 6^2)$.

    a. What is the distribution of $Y = \displaystyle\sum_{i=1}^{16} X_i$ .

    b. Find the constant $c$ such that $P(Y \leq c) = 0.9772$.

7. Let $X$ be a random variable with mean 50 and standard deviation 14. Let $\overline{X}$ be the sample mean of a random sample of size 49 from this distribution. Find $P(48 \leq \overline{X} \leq 54)$.