Probability and Statistics
Spring 2005
Test 4
Name:…………………………………………………………..
25+10+10+10+16+15+5+10 = 101

1 Let $X_1$ and $X_2$ be a random sample of size 2 from $N\left(10,\ 5^2\right)$. Determine

(a) $P\left(X_1 < 20 \text{ and } X_2 > 10\right)$

(b) $E\left(X_1 X_2\right)$

(c ) $Var\left(X_1 X_2\right)$.

(d) $E\left(X_1 + 3X_2\right)$

(e) $Var\left(X_1 + 3X_2\right)$

2 Let $X_1$ and $X_2$ be a random sample of size 2 from a distribution with p.d.f. $f(x) = 6x(1-x),\ 0 < x < 1$. Find the mean and variance of $Y = 2X_1 + X_2$.

(a) $E(2X_1 + X_2)$

(b) $Var(2X_1 + X_2)$

3 Let $X_1$ and $X_2$ be two independent random variables with respective means $\mu_1$ and $\mu_2$, and respective variances $\sigma_1^2$ and $\sigma_2^2$. Show that for real numbers $a_1$ and $a_2$, $Var\left(a_1X_1 + a_2X_2\right) = a_1^2 Var\left(X_1\right) + a_2^2 Var\left(X_2\right)$.

4 Let $X_1$ and $X_2$ be two independent random variables with respective moment generating functions (m.g.f.) $M_{X_1}(t) = e^{10t+18t^2}$ and $M_{X_2}(t) = e^{20t+8t^2}$. Let $Y = 2X_1 + X_2$.

(a) Derive the m.g.f. of $Y$.

(b) Find $P(Y > 64.8)$.

5 Let $X_1,\ X_2, \ldots,\ X_{25}$ be a random sample of size 25 from $N(100,\ 400)$. Find

(a) the distribution of $\bar{X}$. (name, mean, variance)

(b) $P\left(\bar{X} > 110\right)$.

(c ) the distribution of $\sum_{i=1}^{25} X_i$. (name, mean, variance)

(d) $P\left(\sum_{i=1}^{25} X_i > 2300\right)$.

6 Let $X$ be a random variable with mean 100 and variance 196. Let $\bar{X}$ be the sample mean of a random sample of size 49.

(a) What is the approximate distribution of $\bar{X}$? (name, mean, variance)

(b) What result did you use here?

(c ) Find $P\left(\bar{X} > 103\right)$.

7 Let $X_1$, $X_2$, and $X_3$ be a random sample from a Bernoulli distribution with mean 0.3. Find $P\left(X_1 + X_2 + X_3 \le 1\right)$.

8 Let $X_1$, $X_2$, $X_3$, $X_4$, and $X_5$ be mutually independent Poisson random variables having variances 1, 2, 3, 4, and 5 respectively.

(a) Find the m.g.f. of $Y = \sum_{i=1}^{5} X_i$.

(b) Find $P\left(Y = 6\right)$.