Probability and Statistic
Test 4
Spring 06
Name:……………………………………………….
(2+3+3)+8+8+8+8+8+8+8+8+8+20=100

1 If $Z \sim N(0,1)$, then find the following:

(a) $P(|Z| > 2.35)$.

(b) $P(|Z-1| < 0.5)$.

(c) Value of c such that $P(Z^2 > c) = 0.05$

2 If $X \sim N(\mu, \sigma^2)$, then prove that $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$.

3 If $X \sim N(7,4)$, then find the value of c such that $P(X > c) = 0.05$.

4 If $X \sim N(7,4)$, then find the value of c such that $P\left[(X-7)^2 > c\right] = 0.05$.

5 If $X_1 \sim N(7,4)$, $X_2 \sim N(5,4)$, and X_1 and X_2 are independent, then find the value of c such that $P\left[(X_1-7)^2 + (X_2-5)^2 > c\right] = 0.05$.

6 Let $X_1 \sim N(7,4)$, $X_2 \sim N(15,9)$, and X_1 and X_2 are independent. Find $P(2X_1 + X_2 > 24)$.

7 Let $X_1, X_2, \ldots, X_{64}$ be a random sample from a distribution with m.g.f. $M_X(t) = \exp\left(100t + 32t^2\right)$. Find $P\left(\bar{X} > 102.5\right)$. Note: $\exp(x) = e^x$

8 Let $X_1,\ X_2,\ \ldots,\ X_{32}$ be a random sample from a distribution with p.d.f $f(x) = 2x,\ 0 < x < 1$. Find $P\left(\bar{X} > \frac{3}{4}\right)$.

9 Let X be a random variable with mean 50 and standard deviation 14. Let $\bar{X}$ be the sample mean of a random sample of size 49 from this distribution. Find $P\left(48 \le \bar{X} \le 54\right)$.

10 Let $X_1 \sim N\left(10, 3^2\right)$, $X_2 \sim N\left(20, 4^2\right)$, and X_1 and X_2 are independent. Find the moment generating function of $Y = X_1 + X_2$.

11 (This is like your last quiz)

(i) Let $X_1,\ X_2,\ \ldots, X_{16}$ be a random sample from a normal distribution $N(77, 25)$.

What is the distribution of $\bar{X}$? (Name, mean and variance)

(ii) If the m.g.f. of X is $M_X(t) = e^{2t^2}$, find the following:

(a) Distribution, mean and variance.

(b) $P(X > 4)$

(c) x_0 such that $P(X > x_0) = 0.0062$.