Read the questions carefully. In question (1), you have to completely characterize the distribution. For example, a normal distribution is complete when you give the mean and the variance with the name of the distribution. For some other distributions you can give the name and the degrees of freedom.

\[
\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}
\]

Sample mean

\[
S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}
\]

Sample variance

You can use the calculator only for calculations.
(a) If $Z \sim N(0, 1)$, then what is the distribution of $Y = Z^2$?

(b) If $X \sim N(\mu, \sigma^2)$, then what is the distribution of $U = X - \mu$?

(c) If $X \sim N(\mu, \sigma^2)$, then what is the distribution of $V = \frac{2X}{\sigma}$?

(d) If $X_1, X_2, \ldots, X_n$ i.i.d. $\sim N(\mu, \sigma^2)$, then what is the distribution of the sample mean $\bar{X}$?

(e) If $X_1, X_2, \ldots, X_n$ i.i.d. $\sim N(\mu, \sigma^2)$, then what is the distribution of $\frac{(n-1)S^2}{\sigma^2}$?

(f) If $X_1, X_2, \ldots, X_n$ i.i.d. $\sim N(\mu, \sigma^2)$, then what is the distribution of the sum $X_1 + X_2 + \ldots + X_n$?

(g) If $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$, and $X_1$ and $X_2$ are independent, then what is the distribution of $Y = a_1X_1 + a_2X_2$?

(h) If $X_1, X_2, \ldots, X_n$ are a random sample from a distribution with mean $\mu$ and variance $\sigma^2$, then what statistics will give us an approximately standard normal distribution?
Let $Z \sim N(0,1)$. Find the following:

(a) $P(Z > 1.96)$

(b) $P(|Z| > 1.96)$

(c) constant $c$ such that $P(Z > c) = 0.0062$

Prove if $Z \sim N(0,1)$, then $U = Z^2 \sim \chi^2_1$. 
4 If $X \sim N(100, 5^2)$, then find
(a) $P(90 < X < 108)$.
(b) $P(|X - 105| > 8)$. (Note: it is 105 not 100)
(c) a constant $c$ such that $P(|X - 100| > c) = 0.0456$.

5 Let $X_1, X_2, \ldots, X_{100}$ be a random sample from a distribution with moment generating function of $M_X(t) = \exp\left(50t + 50t^2\right)$.
Find $P(48 \leq \bar{X} \leq 51.5)$. 
6 Let $X$ be a random variable with mean 54.03 and standard deviation 6.6. Let $\bar{X}$ be the sample mean of a random sample of size 36 from this distribution. Find $P(53 \leq \bar{X} \leq 56)$.

7 If $Z_1, Z_2, Z_3$ are i.i.d $N(0, 1)$, then prove that the distribution of $U = Z_1^2 + Z_2^2 + Z_3^2$ is $\chi^2_3$. 
8 Let \( X_1, X_2, \ldots, X_{32} \) be a random sample from a distribution with p.d.f
\[
f(x) = 2(1-x), \quad 0 < x < 1.
\]
Find \( P\left( \bar{X} > \frac{3}{4} \right) \).

9 Suppose that the distribution of the weight of a prepackaged “1-pound” bag of carrots is \( N(1.16, 0.07^2) \) and the distribution of the weight of a prepackaged “2-pound” bag of carrots is \( N(2.22, 0.08^2) \). Selecting bags at random, find the probability that the sum of TWO1-pound bags exceeds the weight of ONE 2-pound bag.