Probability and Statistics

Test 4

Spring 2010

Name:……………………………………….

16+10+10+10+8+10+10+10+10+(6 points bonus –no negotiations this time) =100

Read the questions carefully. In question (1), you have to completely characterize the distribution. For example, a normal distribution is complete when you give the mean and the variance with the name of the distribution. For some other distributions you can give the name and the degrees of freedom.

Sample mean $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Sample variance $S^2 = \dfrac{\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2}{n-1}$

You can use the calculator only for calculations.

1 (a) If $Z \sim N(0,\ 1)$, then what is the distribution of $Y = Z^2$?

(b) If $X \sim N\left(\mu,\ \sigma^2\right)$, then what is the distribution of $U = X - \mu$?

(c) If $X \sim N\left(\mu,\ \sigma^2\right)$, then what is the distribution of $V = \dfrac{X}{\sigma}$?

(d) If $X_1,\ X_2,\ldots,X_n \overset{i.i.d.}{\sim} N\left(\mu,\ \sigma^2\right)$, then what is the distribution of the sample mean $\overline{X}$?

(e) If $X_1,\ X_2,\ldots,X_{10} \overset{i.i.d.}{\sim} N\left(\mu,\ \sigma^2\right)$, then what is the distribution of $\dfrac{9S^2}{\sigma^2}$?

(f) If $X_1,\ X_2,\ldots,X_n \overset{i.i.d.}{\sim} N\left(\mu,\ \sigma^2\right)$, then what is the distribution of the sum $X_1 + X_2 + \ldots + X_n$?

(g) If $X_1 \sim N\left(\mu_1,\ \sigma_1^2\right),\ X_2 \sim N\left(\mu_2,\ \sigma_2^2\right)$, and X_1 and X_2 are independent, then what is the distribution of $Y = a_1X_1 - a_2X_2$?

(h) If $X_1,\ X_2,\ldots,X_n$ are a random sample from a distribution with mean μ and variance σ^2, then what statistics will give us an approximately standard normal distribution?

2 If $Z \sim N(0,1)$, then derive the distribution of $Y = Z^2$.

$$f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}; \ -\infty < z < \infty$$

3 If $X \sim N(100, 6^2)$, then find

(a) $P(94 < X < 109)$.

(b) $P(|X - 105| > 8)$. (Note: it is 105 not 100)

(c) a constant c such that $P(|X - 100| > c) = 0.0524$.

4 Let X_1, X_2,..., X_{100} be a random sample from a distribution with moment generating function of $M_X(t) = \exp\left((0)t+2t^2\right)$.

Find $P\left(-0.2 \le \overline{X} \le 0.3\right)$.

5 If $Z \sim N(0,1)$, then find the followings:

(a) $P(Z > 2.35)$

(b) $P(|Z| > 2.35)$

(c) a constant c such that $P(|Z| < c) = 0.8664$

(d) $Z_{0.9772}$

6 Let $X_1,\ X_2, \ldots,\ X_{32}$ be a random sample from a distribution with p.d.f. $f(x) = 2x$, $0 < x < 1$. Find $P\left(\bar{X} > \frac{15}{24}\right)$.

7 Suppose that the distribution of the weight of a prepackaged “1-pound” bag of carrots is $N\left(1.16, 0.07^2\right)$ and the distribution of the weight of a prepackaged “2-pound” bag of carrots is $N\left(2.22, 0.08^2\right)$. Selecting bags at random, find the probability that the sum of TWO 1-pound bags exceeds the weight of ONE 2-pound bag.

8 Let $X_1 \sim N(7,16)$, $X_2 \sim N(15,9)$, and X_1 and X_2 are independent. Find $P(X_1 - X_2 > 2)$. You need to find the distribution of $Y = X_1 - X_2$ first and then $P(Y > 2)$.

9 Let X_1 and X_2 be independent random variable with moment generating functions $M_{X_1}(t) = e^{10t+2t^2}$ and $M_{X_2}(t) = e^{50t+8t^2}$. Find the following if $Y = X_1 + X_2$:

a. $M_Y(t)$

b. $E(Y)$

c. $Var(Y)$