

Time Series Analysis,  
Fall 2008  
Test 2

Name:.....

Do not use computers.

Group work is not allowed.

1 Find the Seasonal Index for July, August, and September of 1994.

Period	Sales	Seasonal Index
1994	January	154
	February	96
	March	73
	April	49
	May	36
	June	59
	July	95
	August	169
	September	210
	October	278
	November	298
	December	245
1999	January	200
	February	118
	March	90
	April	79
	May	78
	June	91
	July	167
	August	169
	September	289
	October	347
	November	375
	December	203



4 Consider the following data:

$X$	$Y$
-4	0.5
-3	1.6
-2	2.3
-1	3.0
0	4.0
1	5.2
2	6.4
3	7.6
4	9.0

- (a) Find the simple linear regression line.
- (b) Construct the ANOVA table.
- (c) Estimate  $\sigma^2$ .
- (d) Find the standard error of the slope.
- (e) Test whether the slope is equal to zero at 0.05 level of significance.
- (f) Find the coefficient of determination.
- (g) Find the correlation between  $X$  and  $Y$ .
- (h) Estimate the fitted values.
- (i) Estimate the errors.
- (j) Forecast the value of  $Y$  for  $X = 5.0$ .
- (k) Find the standard error of the forecast at  $X = 5$ .
- (l) Construct a 95% prediction interval for  $Y$  when  $X = 5.0$ .
- (m) Calculate sample autocorrelations and test for independence of the residuals.
- (n) Plot the residuals versus the fitted values.
- (o) Is the linear model appropriate? If not why?
- (p) Find  $h_{11}$ .
- (q) Find the adjusted  $R^2$ .

5 Consider the following data set.

Y	X1	X2
42	-1	-1
39	-1	-1
48	-1	1
51	-1	1
49	1	-1
53	1	-1
61	1	1
60	1	1

It is given to you that the individual simple linear regression fits are:

a)  $\hat{Y} = 50.375 + 5.375X_1$  and b)  $\hat{Y} = 50.375 + 4.625X_2$ .

Everything you do from this point onwards will be about the following model:

c)  $E(Y) = \beta_0 + \beta_1X_1 + \beta_2X_2$ .

- Estimate the parameters of the multiple regression model  $E(Y) = \beta_0 + \beta_1X_1 + \beta_2X_2$ .
- Find  $h_{11}$ .
- Find the fitted values.
- Estimate the residuals.
- Construct the ANOVA table.
- Estimate  $\sigma^2$ .
- Test whether  $\beta_1=0$  and  $\beta_2=0$  simultaneously at 0.05 level of significance.
- Test whether  $\beta_1=4$  at 0.05 level of significance.
- Predict  $Y$  when  $X_1=0.5$  and  $X_2=0.5$ .
- Construct a 95% prediction interval for  $Y$  when  $X_1=0.5$  and  $X_2=0.5$ .
- Find the adjusted  $R^2$ .
- Find  $Corr(X_1, X_2)$ .
- Discuss multicollinearity for this data. Comment on the parameter estimates of the three models
- Find the studentized residual for the first observation.

6 Consider the following Model:

$$\begin{aligned} Y &= \text{Dependent variable} \\ X &= \text{Independent variable} \\ Z &= \begin{cases} 1 \text{ for Group 1} \\ 0 \text{ for Group 2} \end{cases} \end{aligned}$$

$$XZ = X * Z$$

Fit the model  $E(Y) = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$

Fitted model is

$$\hat{Y} = 33.8383 - 0.1015X + 8.1313Z - 0.0004XZ$$

- (a) Find the regression line for Group 1.
- (b) Find the regression line for Group 2.
- (c) What are you testing with  $\beta_3=0$  versus  $\beta_3 \neq 0$ .
- (d) What are you testing with  $\beta_2=0$  versus  $\beta_2 \neq 0$ .