

Elementary Statistics  
Test 2 preparation

1 Consider the experiment of rolling a **fair** 8-sided die. Then the sample space is  $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{2, 3, 5, 6\}$  and  $C = \{6, 7, 8\}$ .

(i) Draw a Venn diagram.

(ii) Find the following:

(a)  $A \cup B$

(b)  $A \cap B$

(c)  $(A \cap B)'$

(d)  $(A \cup C)'$

(e)  $A \cap B'$

(f)  $A' \cap B'$

(g)  $A' \cup B'$

(h)  $A \cap C'$

(i)  $P[A \cap B]$

(j)  $P[(A \cap B)']$

(k)  $P[A \cap B']$

2 Given  $P(A)=0.59$ ,  $P(B)=0.46$ , and  $P(A \cap B)=0.28$ , draw a Venn diagram, fill in the probabilities associated with the various regions, and thus determine

(a)  $P(A \cup B)$ ;

(b)  $P(A \cap B')$ ;

(c)  $P(A' \cap B')$ ;

(d)  $P(A' \cup B')$ .

3 Check whether the given function can serve as the probability distribution of an appropriate random variable. **Explain your answer.**

(a)

$X$	2	4	6
$f(x)$	0.5	0.3	0.2

(b)

$X$	2	4	6
$f(x)$	- 0.1	0.6	0.5

- 4 Let a random variable  $X$  have a binomial distribution with  $n = 10$  and  $p = 0.2$ . Find the following. Hint: Use the tables.
- $P ( X = 2 )$
  - $P ( X \text{ is at least } 5 )$
  - $P ( X \text{ is less than } 2 )$
  - $P ( X \text{ is more than } 4 )$
  - $P ( X \text{ is at most } 2 )$
  - Mean and variance.
- 5 Childhood lead poisoning is a public health concern in the United States. In a certain population one in ten children has a high blood level of lead. In a randomly chosen group of 15 children from this population, what is the probability that
- none has high blood level of lead?
  - at most 3 have high blood level of lead?
  - less than 3 have high blood level of lead?
  - more than 4 have high blood level of lead?
- 6 Find the mean, variance and the standard deviation of the following distribution.

$X$	1	2	3
$f(x)$	0.2	0.6	0.2

- 7 Consider the continuous uniform distribution  $f(x) = \frac{1}{2}$ ,  $0 \leq x \leq 2$ . Find the following probabilities.
- $P(0 < x < 0.7)$
  - $P(x \geq 0.4)$

8 Let the random variable  $Z$  have a standard normal distribution. Find the following probabilities. **Also draw graphs.**

- (a)  $P(Z < 2.5)$
- (b)  $P(-2.5 < Z < 2.5)$
- (c)  $P(1.23 < Z < 2.34)$
- (d)  $P(Z > 2.34)$

9 If  $z_\alpha$  denotes the value of  $z$  for which the area under the standard normal curve to its right is equal to  $\alpha$ , find

- (a)  $z_{0.025}$ ;
- (b)  $z_{0.005}$ ;
- (c)  $z_{0.05}$ .

10 Drivers who are members of a union earn an average of \$20.00 per hour. Assume that available data indicate wages are normally distributed with a standard deviation of \$2.25.

- (a) What is the probability that wages are between \$15.50 and \$24.50 per hour?
- (b) What is the probability that the wages are less than \$15.00 per hour?
- (c) Find the wage  $w_0$  such that only 2.5% of the union drivers earn more than  $w_0$ .

11 Let a random variable  $X$  have a normal distribution with mean 10 and standard deviation 2. Find the following probabilities:

**Draw graphs.**

- (a)  $P(X \text{ is at least } 13)$
- (b)  $P(X \text{ is less than } 7)$
- (c)  $P(X \text{ is more than } 14)$
- (d)  $P(X \text{ is at most } 15)$

- 12 Sample space is all the possible outcomes of an experiment. (T, F).
- 13 Probabilities are real numbers between  $-1$  and  $1$ . (T, F)
- 14  $P(S) = S$ . (T, F)
- 15 For any two events  $A$  and  $B$ ,  $P(A \cup B) = P(A) + P(B)$ . (T, F)
- 16  $\mu$  is the symbol for sample mean. (T, F)
- 17 An easy way to find  $\mu$  is to use the formula  $\mu = np$ . (T, F)
- 18 Normal distribution has a symmetric distribution. (T, F)
- 19 If  $A$  and  $B$  are mutually exclusive sets (events), then  $A \cup B$  is an empty set (event). (T, F)
- 20 Normal curves are symmetric about the standard deviation. (T, F)
- 21 Area under the standard normal curve is one unit. (T, F)
- 22 Area under the curve of a normal distribution with mean  $10$  and standard deviation  $2$  is not equal to one. (T, F)
- 23 What are the basic rules of probability?
- 24

	$A$	$A'$	Total
$B$	168	172	
$B'$	57	43	
Total			

Find  $P(B)$ ,  $P(A \cap B)$ , and  $P(A|B)$ .

25 Match the following symbols with the definitions.

- |     |                                    |                  |
|-----|------------------------------------|------------------|
| (a) | Sample mean                        | $A'$ _____       |
| (b) | Sample variance                    | $s$ _____        |
| (c) | Population mean                    | $\sigma^2$ _____ |
| (d) | Population variance                | $s^2$ _____      |
| (e) | Median                             | $\bar{X}$ _____  |
| (f) | Sample standard deviation          | $\sigma$ _____   |
| (g) | Population standard deviation      | $A \cap B$ _____ |
| (h) | Intersection of events $A$ and $B$ | $\mu$ _____      |
| (i) | Union of events $A$ and $B$        |                  |
| (j) | Complement of event $A$            |                  |

26 Given the **mutually exclusive** events  $Y$  and  $Z$ , for which  $P(Y) = 0.4$  and  $P(Z) = 0.3$ , find

- (a)  $P(Y')$ ;
- (b)  $P(Y' \cap Z)$ ;
- (c)  $P(Y' \cap Z')$ ;
- (d)  $P(Y \cap Z)$ ;
- (e)  $P(Y \cup Z)$ .

28 What are the four basic rules of probability?

29 Convert probabilities to odds and odds to probabilities.

- (a)  $p = \frac{7}{11}$
- (b)  $a$  to  $b$  is 13 to 17.

30 Fill in the blanks of the following tables.

Probability	
$x$	$f(x)$
0	0.0532
1	0.2300
2	0.3738
3	0.2699
4	0.0731

Cumulative Probability	
$x$	$F(x)$
0	
1	
2	
3	
4	

31 Fill in the blanks of the following tables.

Cumulative Probability	
$x$	$F(x)$
0	0.1296
1	0.4752
2	0.8208
3	0.9744
4	1.0000

Probability	
$x$	$f(x)$
0	
1	
2	
3	
4	

33 Random variable  $X$  have a Binomial distribution with mean 2.8 and  $p=0.7$ . Find the variance and standard deviation.

34 Suppose that 5 cards are dealt, without replacement, from a standard deck of 52 cards. What is the probability that 5 cards will include,

- (a) exactly one spades?
- (b) no spades?
- (c) two spades?
- (d) at most two spades?
- (e) at least two spades?

35 Among 15 applicants for sales positions in a corporation, 8 have college degrees and 7 do not have college degrees. If 4 are randomly selected for interviews, what is the probability that all have college degrees?