

Elementary Statistics

November 15, 2010

Test 3

Name:.....

$$(1+2+10)+3+10+10+10+4+(7+1+1)+9+9+9+5+9+4=100$$

1 Consider a random variable  $X$  with mean 100 and standard deviation 2.0. Suppose you take samples of size 64. Find the following:

(a) Mean of the sampling distribution.

(b) Standard error of the mean.

(c)  $P(99.60 < \bar{X} < 100.50)$

2 Let the random variable  $X$  have a distribution with mean 67 and standard deviation 7.07. Take a sample of size 50. Find the standard error of the mean if the population size is 10000

- 3 A survey reported that 37% of college freshman carry a credit card balance from month to month. Assume the sample size is 1000. Construct a 95% confidence interval for the true proportion of college freshman who carry a credit card balance from month to month. Think carefully, I have given you all the numbers you need. May be in a different way!
  
- 4 A survey on the working women of North America was conducted by the Clinique unit of Estee Lauder Cosmetics. Of the 1000 women surveyed 450 (45%) believed that companies should hold positions for those on maternity leave for more than six months. Find the maximum error of the estimate for the true proportion of women who believe that the companies should hold positions for those on maternity leave for more than six months. Use 0.99 confidence level.
  
- 5 A political pollster wants to estimate the proportion of voters who will vote for the Republican candidate in a presidential campaign. The pollster wants to have 90% confidence that her prediction is correct to within 0.04 of the true population proportion. How large a sample does he need?
  
- 6 Five cards are drawn from a well shuffled pack of cards. What is the probability that Heart King will be in the sample? (Set up. Do not simplify.

- 7 A quality control manager wants to estimate the mean life of light bulbs produced in a batch to within 20 hours with 95% confidence. He knows that the life distribution is approximately normal with a standard deviation of 100 hours.
- (a) How large a sample does he need?
  
  - (b) If he increases the level of confidence to 99% keeping the other numbers constant will the required sample size increase? Yes/No
  
  - (c) If he increases the margin of error to 30 hours keeping the other numbers constant will the required sample size increase? Yes/No
- 8 A soda bottling machine is set to fill bottles to little more than two liters with a standard deviation of 0.05 liters. To test whether the true mean is more than 2 liters a group of students measures 25 bottles and found a sample mean of 2.08 liters with a standard deviation of 0.048 liters. Construct a 90% confidence interval for the true average volume of a filled bottle. Assume the volume in the soda bottles follow a normal distribution.
- 9 The amount of juices from oranges from a large grove has a normal distribution. If a sample of 100 oranges produces a sample mean of 4.70 ounces with a standard deviation of 0.40 ounces, then find a 90% confidence interval for the true mean amount of juices from oranges from this grove.

10 A specialist in precision machinery wants to estimate the mean expansion of certain pistons (in inches) on the basis of a sample of 42 of these pistons. The expansion is caused by the heat generated after the engines have been started, and it is known that  $\sigma = 0.02$  inch. If the specialist considers the data to constitute a random sample, what can he assert with probability 0.95 about the maximum error if he uses the mean of his sample as an estimate of the actual mean expansion of such pistons?

11 Fill in the blanks.

Central Limit Theorem

If \_\_\_\_\_ is the mean of a random sample of size  $n$  from an infinite population, with the mean \_\_\_\_ and the standard deviation \_\_\_\_, and \_\_\_\_ is large, then \_\_\_\_\_ has approximately the **standard normal distribution**.

12 A random sample of 18 Danish pastries produced by a major bakery products manufacturer has a mean of 190 milligrams of sodium per pastry, with a standard deviation of 10 milligrams per pastry. If 190 milligrams is used as an estimate of the actual mean of the population of Danish pastries produced by this manufacturer, what can we assert about the maximum error with 95% confidence?

13 This is a challenging question (4 bonus points)

If a 95% confidence interval of a population mean is given by (7.8, 9.4), what is the sample mean used to calculate this confidence interval? Using the same information, if you calculated the maximum error with 95% confidence, what will be the value of  $E$ ?