

# Elementary Statistics

## Test 3

### Preparation

$$m_{\bar{x}} = m \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}} \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$z = \frac{x - m}{s} \qquad z = \frac{\bar{x} - m}{s/\sqrt{n}}$$

$$E = z_{\alpha/2} \frac{s}{\sqrt{n}} \qquad E = t_{\alpha/2, (n-1)} \frac{s}{\sqrt{n}} \qquad E = z_{\alpha/2} \sqrt{\frac{\frac{x}{n} \left(1 - \frac{x}{n}\right)}{n}}$$

$$n = \left( \frac{z_{\alpha/2} s}{E} \right)^2 \qquad n = p(1-p) \left( \frac{z_{\alpha/2}}{E} \right)^2 \qquad n = \frac{1}{4} \left( \frac{z_{\alpha/2}}{E} \right)^2$$

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} \qquad \bar{x} \pm t_{\alpha/2, (n-1)} \frac{s}{\sqrt{n}} \qquad \frac{x}{n} \pm z_{\alpha/2} \sqrt{\frac{\frac{x}{n} \left(1 - \frac{x}{n}\right)}{n}}$$

- 1 Consider a random sample of size  $n=64$  from a population with mean  $\mu=10$  and variance  $\sigma^2=16$ . Fill in the following blanks.
- $\bar{X} = \dots\dots\dots$  ( Give the name here)
  - $\bar{X}$  is an estimate for  $\dots\dots$  (Give a symbol here)
  - For large  $n$ , distribution of  $\bar{X}$  is  $\dots\dots\dots$   
( Give the name of the distribution here)
  - Mean of the sampling distribution  $\mu_{\bar{X}} = \dots\dots\dots$  ( Give a numerical answer here)
  - Standard error of the mean is  $\dots\dots\dots$  (Give a numerical answer with calculations here)
  - $P(\bar{X} \leq 11) = \dots\dots\dots$  (Requires some work)
- 2 A biologist wants to estimate the mean weight of a certain kind of animal. He knows that the standard deviation of their weights is 2.5 ounces and decides to use a random sample of size 100.
- What is the standard error of the mean?
  - What can be said with probability 0.95 about the maximum error?
- 3 A large hospital finds that in 50 randomly selected days it had, on average, 96.4 patient admissions per day with a standard deviation of 12.2. Construct a 98% confidence interval for the actual daily average number of hospital admissions.
- 4 Suppose we want to estimate the average IQ score of the elementary statistics students at PSU. How large a sample must be selected if we want 95% confidence that the sample mean is within 3 IQ points of the population mean? Assume that  $\sigma = 15$ .
- 6 A sample of 6 adult elephants had an average weight of 12,200 pounds, with a sample standard deviation of 200 pounds. Find a 95% confidence interval of the true mean.

- 7 In a sample survey 150 of 500 persons interviewed in a large city said that they shop in the downtown area at least once a week.
- (a) Construct a 90% confidence interval for the corresponding true proportion.
  - (b) What can we say with 90% confidence about the maximum error if we use the sample proportion to estimate the true proportion? (That is, find the maximum error.)
- 8 Suppose that we want to estimate what proportion of the adult population of the US has high blood pressure, and we want to be 95% sure that the error of our estimate will not exceed 0.02. How large a sample will we need if
- (a) We have no idea what the true proportion might be;
  - (b) We know that the true proportion is 0.10?
- 9 In 10 test runs, an experimental engine consumed on the average 12.9 gallons of gasoline per minute with a standard deviation of 1.5 gallons. What can we assert with 95% confidence about the maximum error if we use 12.9 as an estimate of the true average gasoline consumption of the engine?
- 10 Given that the population of women has normally distributed weights with a mean of 143 lb and a standard deviation of 29 lb (based on the National Health Survey), find the probability that
- (a) if a woman is randomly selected, her weight is greater than 150 lb.
  - (b) If 36 different women are selected, their mean weight is greater than 150 lb.
- 11 Define the following symbols.

$\bar{X}$ ,  $n$ ,  $N$ ,  $S$ ,  $s$ ,  $m_{\bar{X}}$ ,  $s_{\bar{X}}$ ,  $E$ ,  $p$ ,  $\hat{p}$

- 12 A New York Times poll on women's issues interviewed 1025 women and 472 men randomly selected from the US. The poll announced a margin of error  $\pm 3$  percentage points for 95% confidence in conclusions about women. The margin of error for results concerning men was  $\pm 4$  percentage points.
- Show the calculations to get margins of error for men ( $\pm 4$ ) and women ( $\pm 3$ ).
  - Why does the margin of error for results concerning men is larger than that for women?
- 12 Suppose a simple random sample of size 200 is selected from a population with  $\sigma = 10$ . Find the value of the standard error of the mean if the population size is
- infinite.
  - $N = 5000$ .
- 13 Fill in the blanks. **Central Limit Theorem**  
If \_\_\_\_\_ is the mean of a random sample of size  $n$  from an infinite population, with the mean \_\_\_\_ and the standard deviation \_\_\_\_, and \_\_\_\_ is large, then \_\_\_\_\_ has approximately the standard normal distribution.
- 14 Fill in the blanks. **Central Limit Theorem**  
If \_\_\_\_\_ is the mean of a random sample of size  $n$  from an infinite population, with the mean \_\_\_\_ and the standard deviation \_\_\_\_, and \_\_\_\_ is large, then  $\bar{X}$  has approximately \_\_\_\_\_ distribution with mean of the sampling distribution \_\_\_\_\_ and the standard error of the mean \_\_\_\_\_.
- 15 Adam, Ben, Charles, Doug, Edward, Fred, George, Harry, and Ian are 8 friends. How many different samples of size 3 can be drawn from this population of size 8?
- 16 Using the information in problem 15, find the probability that Adam will be in the sample.
- 17 Using the information in problem 15, find the probability that both Adam and Ben will be in the sample.