1 Fill in the blanks.

Definition: The probability mass function $f(x)$ of a discrete random variable $X$ is a function that satisfies the following properties:

(a) 

(b) 

(c) 

2 For the discrete uniform distribution prove the following.

(a) $E(X) = \frac{m+1}{2}$.

(b) $Var(X) = \frac{m^2 - 1}{12}$. 

A bowl contains 4 white balls and 6 black balls. Three balls are drawn without replacement. Let $X$ be the number of white balls drawn. Find the distribution of $X$. Set up do not simplify.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$f(x)$</th>
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(a) If $f(x) = c \left( \frac{1}{2} \right)^x ; x = 1, 2, 3, ..., 10$, find the value of $c$.

(b) If $f(x) = \frac{c}{2^{|x|}} ; x = \pm 1, \pm 2, \pm 3, ...$, find the value of $c$. 
If \( f(x) = \left( \frac{1}{3} \right)^x \left( \frac{2}{3} \right)^{1-x}; x = 0,1 \), find the following.

(a) \( E(X) \).

(b) \( Var(X) \).

(c) \( E(3X + 2) \).

(d) \( Var(3X + 2) \).

(e) \( E[Var(3X + 2)] \).

(f) \( Var[E(3X + 2)] \).

Consider the experiment of tossing a fair coin twice. Let the random variable \( X \) be the number of heads.

(a) Write down the sample space.

(b) Find the probability mass function of \( X \).

(c) What is the distribution of \( X \)? Give the parameter values.
Let $f(x) = q^{x-1} p$; $x = 1, 2, 3, \ldots$.

(a) Show that $\sum_{x=1}^{\infty} f(x) = 1$.
(b) Show that $E(X) = \frac{1}{p}$.
(c) Show that $E[\max(2, X)] = p + \frac{1}{p}$.

Let $X \sim Bin(n = 2000, p = 0.001)$. Use the Poisson approximation to find $P(X > 1)$. Do not simplify the answer.
9 If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it? Set up do not simplify.

10 Draw the cumulative distribution function $F(x)$ and find $f(x)$ using the following $F(x)$.

$$F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
0.2 & \text{if } 0 \leq x < 1 \\
0.5 & \text{if } 1 \leq x < 2 \\
1.0 & \text{if } x \geq 2 
\end{cases}$$
(a) Derive the moment generating function for the Poisson distribution.

(b) Find the mean and variance by taking the derivatives of the m.g.f.

(c) Define the function $R_X(t)$ as the natural log of the m.g.f. and find the mean and variance by taking the derivatives of this function.