Let \( F(x) = \begin{cases} 
0 & \text{if } x < -2 \\
\frac{(2+x)^2}{8} & \text{if } -2 \leq x < 0 \\
1 - \frac{(2-x)^2}{8} & \text{if } 0 \leq x < 2 \\
1 & \text{if } x \geq 2 
\end{cases} \)

Find the probability density function (p.d.f.) of \( X \) and plot it.

5pts
Let the p.d.f. of $X$ be $f(x) = 2x$, $0 < x < c$. Find the following:

(a) constant $c$.
(b) mean of the distribution.
(c) Variance of the distribution.
(d) median.
(e) cumulative distribution function (c.d.f.) of $X$.

20pts
3. Let $X \sim \text{Uni}(-a,a)$.

(a) Show that $E(X^r) = 0$ when $r$ is odd.
(b) Find $E(X^r)$ when $r$ is even. Simplify the answer.
(c) Find the mean and the variance using (a) and (b). Do not use the formulas.

10pts
Let $f(x) = 2e^{-2x}$, $x > 0$. Find the following:

(a) $P(X > 5)$.
(b) $P(X > 12 \mid X > 7)$.
(c) Plot the function $g(x) = \max(3, x)$ for $x > 0$ and evaluate $E[g(x)]$.
(d) $P(X > \text{median})$.
(e) $P(X = 5)$.

20pts
5 Let $X \sim N\left(10, 5^2\right)$. If $aX + b \sim N(0, 1)$, find the values of $a$ and $b$. 
5 pts

6 If $X \sim N(50, 5^2)$, find a constant $c$ such that $P\left(|X - 50| > c\right) = 0.1336$. 
8 pts
7. If $X \sim N\left(10, 2^2\right)$. Find the following:

(a) $P(X > 13)$.
(b) $P(6 < X < 12)$.
(c) $P\left( X + \frac{72}{X} > 18 \right)$.

12 pts
Let the p.d.f. of $X$ is $f(x) = 2x$, $0 < x < 1$. Find the p.d.f of $Y = 3X + 6$.
Let $X \sim \text{Uni}(0, 3)$. Consider the transformation $Y = (X - 1)^2$.

(a) Plot $Y$ against $X$ for $0 \leq X \leq 3$.
(b) What is the support of $Y$?
(c) Find the values of $Y$ for which the transformation is one to one.
(d) Find the p.d.f. of $Y$. 

10 pts