Let the p.d.f. of $X$ be $f(x) = \begin{cases} x + 1, & -1 < x < 0, \\ 1, & 0 \leq x < 0.5. \end{cases}$

(a) Find the cumulative density function (c.d.f.) of $X$.

(b) Sketch the graph of the p.d.f. of $X$.

(c) Sketch the graph of the c.d.f. of $X$.

(d) Find the mean of the distribution.

(e) Find the variance of the distribution.
Let the p.d.f. of $X$ be $f(x) = 4e^{-4x}$, $x \geq 0$.

(a) Derive the moment generating function.
(b) Find the c.d.f. of $X$.
(c) Evaluate the integral $E(X^2) = \int_0^\infty 4x^2 e^{-4x} \, dx$.
(d) Prove that the median is $0.25\ln 2$. 
3. (a) Prove that, if $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X - \mu}{\sigma} \sim N(0,1)$.

(b) If the m.g.f. of $X$ is $M_X(t) = e^{2t^2}$, then find the following.
   
   (i) $P(X > 2.35)$.

   (ii) $P(X < -1.35)$.

   (iii) $P(0 < X < 2.00)$.

   Draw a graph and the shade the area of interest.
(a) Let the p.d.f. of $X$ be given by the following table.

<table>
<thead>
<tr>
<th>$X$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Find the p.d.f. of $Y = X^2$. (Make a probability table for $Y$.)

(b) Let the c.d.f. of $X$ be $F(x) = 1 - e^{-x^2/2}$, $x > 0$.

(i) Find the p.d.f. of $X$.
(j) Find the p.d.f. of $Y = X^2$. 