1. Let $X_1$ and $X_2$ be independent random variables with p.d.f.'s $f_1(x_1) = 2x_1$, $0 < x_1 < 1$ and $f_2(x_2) = 4x_2^3$, $0 < x_2 < 1$, respectively. Compute

(a) $P(0.5 < x_1 < 1 \text{ and } 0.4 < x_2 < 0.8)$.

(b) $E[X_1^2 X_2^3]$.

2. Let $X_1, X_2, X_3$, and $X_4$ be a random sample from Pois(2). Let $Y = \sum_{i=1}^{4} X_i$.

(a) Find the moment generating function of $Y$ and decide the distribution of $Y$ by observation.
Let $X_1$ and $X_2$ be a random sample from a distribution with p.d.f $f(x) = 12x^2(1-x)$, $0 < x < 1$. Let $Y = X_1 + 2X_2$.

(20 pts)

(a) Find the mean of $Y$.

(b) Find the variance of $Y$.

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4 Prove the following theorem.

(10 pts)

If $X_1$, $X_2$, ..., $X_n$ are observations of a random sample of size $n$ from the normal distribution $N(\mu, \sigma^2)$, then the distribution of the sample mean $\bar{X}$ is $N\left(\mu, \frac{\sigma^2}{n}\right)$. Hint: First find $M_X(t)$.
Let $X_1, X_2, \ldots, X_{16}$ be a random sample from a normal distribution $N(77, 25)$. Compute

(a) $P(75 < \bar{X} < 79.5)$.

(b) $P\left(1200 < \sum_{i=1}^{16} X_i < 1272\right)$.

Let $X_1, X_2, \ldots, X_{48}$ be a random sample from $\text{Unif}(0, 6)$.

(a) Find the approximate distribution of $\bar{X}$.

(b) Compute the approximate probability of $\bar{X} > 3.4$. 
Let $X_1$ and $X_2$ be two independent random variables with respective means $\mu_1 = 4$ and $\mu_2 = 5$ and variances $\sigma_1^2 = 4$ and $\sigma_2^2 = 9$. Find the mean and variance of $Y = X_1X_2$. 

(10 pts)