Let $X_1$, $X_2$, $X_3$, and $X_4$ be a random sample from a geometric distribution with p.d.f. $f(x) = (0.3)^x - 1 0.7$, $x = 1, 2, \ldots$.

(a) Compute $P(X_1 = 1, X_2 = 3, X_3 = 1, X_4 = 2)$.

(b) Find the distribution of $Y = X_1 + X_2 + X_3 + X_4$.

(c) Determine $P(X_1 + X_2 + X_3 + X_4 = 15)$.(Set up, do not simplify)

(d) If $Y = \min(X_1, X_2, X_3, X_4)$, find $P(Y \geq 3)$. 

Let $X_1$, $X_2$ and $X_3$ be three mutually independent random variables with $E(X_1) = 5$, $E(X_2) = 6$, $E(X_3) = 2$, $Var(X_1) = 16$, $Var(X_2) = 4$, and $Var(X_3) = 9$. Find

(a) $E(2X_1 + X_2 + 3X_3)$.

(b) $Var(2X_1 + X_2 + 3X_3)$.

(c) $E(X_1X_2X_3)$.

(d) $E(X_1^2X_2^2X_3^2)$.

(e) $Var(X_1X_2X_3)$. 
3. (a) Prove the following theorem.

If $X_1, X_2, \ldots, X_n$ are $n$ mutually independent normal variables with means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, respectively, then the linear function $Y = \sum_{i=1}^{n} c_i X_i$ has the normal distribution

$$N\left( \sum_{i=1}^{n} c_i \mu_i, \sum_{i=1}^{n} c_i^2 \sigma_i^2 \right).$$

(b) Let $X_1, X_2$, and $X_3$ be $N\left(60, 3^2\right)$, $N\left(48, 4^2\right)$, and $N\left(51, 2^2\right)$, respectively. Find $P\left(\frac{X_1 + X_2}{2} > X_3\right)$. 
(a) State the Central Limit Theorem. I just want you to give me the correct idea of the theorem.

(b) Let \( X_1, X_2, \ldots, X_n \) be a random sample of size 36 from an infinite population with mean 100 and variance 25. Find

(i) \( E(\bar{X}) \).

(ii) \( Var(\bar{X}) \).

(iii) \( P(X_i > 101) \) for any \( i = 1, 2, \ldots, n \).

(iv) \( P(\bar{X} > 101) \).