A fair six-sided die is rolled once. Let $A = \{1, 3, 5\}$ and $B = \{3, 6\}$. Find the following. (15 points)

(a) $P(A)$
(b) $P(B)$
(c) $P(A \cap B)$
(d) $P(A \cup B)$
(e) $P(A' \cap B)$
(f) $P(A' \cup B)$
(g) $P(A' \cup B')$
(h) $P(A | B)$
(i) $P(A' | B)$
(j) $P(A | A \cup B)$
(k) $P(A \cap B | A)$
(l) $P(A \cap B | A \cup B)$

Give a set theoretic proof to one of the following theorems. (12 points)

(a) If $A$ and $B$ are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
(b) If $A$ and $B$ are independent, then so are $A$ and $B'$. 
3 Suppose that 12 six-sided balanced dice are to be rolled. Find the probability that each of the six numbers will appear twice. Simplify. (12 points)

4 In a pack of playing cards, there are 13 different face values. Namely, Ace, 2, 3, ...., J, K, and Q. (14 points)
In a poker hand, find the probability of a four of a kind (four cards of equal face value and one card of a different value).

5 A small grocery store had 10 cartons of milk, three of which were sour. If you are going to buy the sixth carton of milk sold that day at random, compute the probability of selecting the third carton of sour milk sold that day. (12 points)

6 Each of three football players will attempt to kick a field goal from the 25-yard line. Let $A_i$ denote the event that the field goal is made by player $i$, $i=1, 2, 3$. Assume that $A_1$, $A_2$, and $A_3$ are mutually independent and that $P(A_1)=0.5$, $P(A_2)=0.7$, $P(A_3)=0.6$. (12 points)
(a) Compute the probability that exactly one player is successful.
(b) Compute the probability that exactly two players make a field goal.

7 Bean seed from supplier A have a 90% germination rate and those from supplier B have an 80% germination rate. A seed packing company purchases 45% of their bean seeds from supplier A and 55% from supplier B and mixes these seeds together. (12 points)
(a) Find the probability that a seed selected at random from the mixed seeds will not germinate, say $P(G')$.
(b) Given that a seed does not germinate, find the probability that the seed was purchased from supplier B.

8 Consider two events $A$ and $B$ such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{2}$. Determine the value of $P(B \cap A')$ for each of the following conditions. (12 points)
Draw Venn diagrams whenever possible.
(a) $A$ and $B$ are disjoint (mutually exclusive);
(b) $A \subset B$ ;
(c) $P(A \cap B)=\frac{1}{8}$ ;
(d) $A$ and $B$ are independent.