Let \( X_1, X_2, \) and \( X_3 \) be mutually independent Poisson random variables with respective means 2, 3, and 5.

(a) **Derive** the moment generating function of \( Y = \sum_{i=1}^{3} X_i \).

(b) Find \( P(Y > 1) \).

Let \( X_1, X_2, \) and \( X_3 \) be mutually independent Poisson random variables with respective means 2, 3, and 5 and variances 4, 9, and 16. Find

(a) \( \text{Var}(3X_1 + 2X_2 + X_3) \).

(b) \( E(X_1X_2X_3) \).

(c) \( \text{Var}(X_1X_2X_3) \).

Let \( X_1 \) and \( X_2 \) be a random sample of size 2 from a distribution with the probability density function \( f(x) = 12x^2(1-x), \ 0 < x < 1 \). Let \( Y = 2X_1 + X_2 \).

(a) Find \( E(Y) \).

(b) Find \( \text{Var}(Y) \).

Prove the following theorem.

If \( X_1, X_2, \ldots, X_n \) are \( n \) mutually independent normal variables with means \( \mu_1, \mu_2, \ldots, \mu_n \) and variances \( \sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2 \), respectively,

then the linear function \( Y = \sum_{i=1}^{n} c_i X_i \) has the normal distribution \( \mathcal{N}\left( \sum_{i=1}^{n} c_i \mu_i, \sum_{i=1}^{n} c_i^2 \sigma_i^2 \right) \).

(b) Let \( X_1, X_2, \) and \( X_3 \) be \( \mathcal{N}\left(60, 3^2\right), \mathcal{N}\left(48, 4^2\right), \) and \( \mathcal{N}\left(51, 2^2\right) \), respectively. Find \( P\left(\frac{X_1 + X_2}{2} > X_3\right) \). Assume \( X_1, X_2, \) and \( X_3 \) are mutually independent.
Let $X_1, X_2, \ldots, X_{75}$ be a random sample from $\text{Uni}(-1,1)$.

(a) Find the approximate distribution of $\bar{X}$.

(b) Compute the approximate probability of $\bar{X} > \frac{2}{15}$.

Prove without much work.

Let $X_1$ and $X_2$ be a random sample of size 2 from $f(x) = 3e^{-3x}, x \geq 0$.

Find $E(X_1 X_2^2)$.

Let $X_1, X_2$, and $X_3$ be a random sample from $f(x) = \frac{e^{-3} 3^x}{x!}, x = 0, 1, 2, \ldots$

Find $E[(X_1 - 3) X_2^2 (X_3 - 3)^2]$.

The joint discrete distribution $f(x, y)$ is given below. Show that $X$ and $Y$ are not independent.

$$
\begin{array}{c|cc}
X \rightarrow & 1 & 2 \\
Y \downarrow & \ & \\
1 & 0.0 & 0.3 \\
2 & 0.3 & 0.4 \\
\end{array}
$$